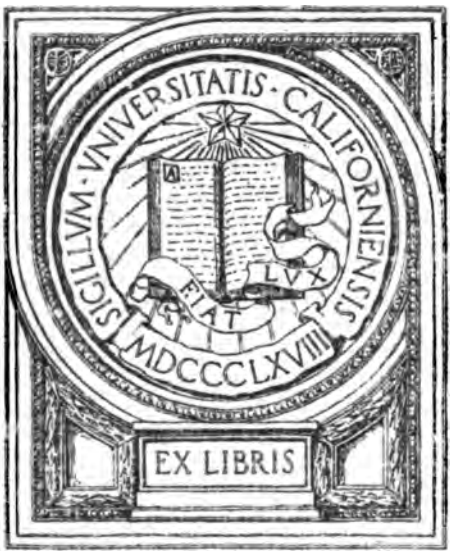
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NOTES

ON

Heat, Steam and the Steam Engine,

FOR THE USE OF STUDENTS

OF THE

RENSSELAER POLYTECHNIC INSTITUTE.

BY

D. M. GREENE,

DIRECTOR.

TROY, N. Y.:
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PREFACE TO SECOND EDITION.

The present edition is, as the first was, published in haste. Many typographical and other errors have, however, been discovered and corrected; and it is believed that the "*Notes*," as now presented, are reasonably free from error. Additional notes have been introduced, in the Appendix, relating to the efficiencies of various fuels, to certain conditions which affect the economy of the steam-engine, and to the new vessels which constitute the nucleus of the modern Navy of the United States. Tables have also been introduced, by means of which computations of power and speed of steam vessels may be facilitated.

In regard to the imperfection of, and want of proportion in, the diagrams, it is to be remembered that these are from drawings made by several students, of the class of 1886, from hastily executed black-board sketches. These will be pointed out and explained, orally, during the progress of the course.

D. M. G.

TROY, N. Y., Jan. 24th, 1888.

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NOTES ON THE STEAM ENGINE.

I. Elementary Considerations—Combustion—Heat.

1. *Definition.*—The Steam Engine—including the furnace, boiler and chimney—is a machine in which heat is generated and converted into work. Heat is generated by the combustion of fuel—either solid, liquid or gaseous, and is transmitted through the material of the boiler, to the water, which is first heated, to the boiling point, and is then converted into steam.

2. *Steam* is the medium or agency by which the heat is converted into work: it passes through the steam pipe, to the steam cylinder, where, by its elastic force, it communicates motion to the piston and thus performs work. A measure of the work performed is the quantity of heat which disappears during the operation.

3. *The Steam Engine*, as a whole, consists, essentially, of a furnace, chimney, boiler, steam-pipe and engine proper, together with their necessary appurtenances.

4. *Unit of Heat.*—The unit of heat is the heat necessary to raise the temperature of a pound of water one degree (39.1° to 40.1°) Fahrenheit.

5. *Mechanical Equivalent*—The mechanical equivalent of a unit of heat, as ascertained by Joule and others—sometimes designated a "Joule"—is, in round numbers, a force of 772 pounds, exerted through a space of one foot, or 772 foot-pounds.

This is called the English unit.

The French unit of heat is the heat necessary to raise the temperature of one kilo. of water from 3.94° to 4.94° Cent.; it is

equivalent to 3.968 English thermal units and is designated as a *Calorie*.

6. *Illustration.*—The mechanical equivalent of the heat necessary to raise the temperature of 1 lb. of water from 50° Fahr. to 212° Fahr. is 772 (212 − 50) = 125046 foot-pounds. If this heat be applied in one minute, the equivalent, in *power*, will be

$$\frac{125046}{33000} = 3.79 \text{ horses.}$$

7. *Work or Power not Wholly Available.*—The work, or power, due to the heat generated, and applied to the water, cannot be wholly utilized, for the reason that various losses occur between the furnace and the steam cylinder, and because the steam, after performing its work, still retains the larger part of its original heat.

8. *The Horse-Power.*—This is the mean power of a London draught horse, as ascertained by James Watt, and is equivalent to 33,000 pounds raised one foot in one minute; or to 550 pounds raised one foot in one second.

The standard horse-power exceeds, materially, the power of the average American horse.

9. *The Fuel.*—Inasmuch as the power developed by the steam engine depends upon the heat generated and upon the economy with which it is applied, it is obviously proper, in the study of the engine, to begin with a consideration of the subject of *fuel* and of the quantities of heat due to the combustion of a unit of each of the more common fuels.

10. *Combustion.*—This, chemically considered, is the result of the combination of oxygen with a combustible—or fuel. The combustible elements of all fuels are, of course, carbon and hydrogen.

The air supplies the oxygen. The products of combustion are therefore carbonic acid gas (carbon dioxide), carbonic oxide (carbon monoxide), vapor of water, nitrogen and air.

11. *Heat Produced.*—This is proportional to the quantity of oxygen which combines with a unit of the combustible.

The following table gives the number of heat units resulting from the combustion of one pound of each of the principal fuels:

Hydrogen Gas	62000	units.
Carbon, $C\,O_2$	14500	"
" $C\,O$	4400	"
Liquid Hydro-Carbons from	19000	"
to	21000	"
Charcoal, wood	13500	"
" peat	11600	"
Coke, good	13620	"
" middling	12760	"
" bad	11890	"
Coal, anthracite	15225	"
" dry bituminous	15370	"
" " "	14860	"
" " "	14790	"
" " "	13375	"
" cannel	15080	"
Peat, dry	9660	"
" 25% moisture	7000	"
Wood, dry	7245	"
" 20% moisture	5600	"

12. *Examples.*—Let it be required to determine the number of heat-units due to the combustion of a pound of anthracite coal, in which

Fixed carbon, C	$= 88.71\%$.
Volatile combustible, $C\,H_2$	$= 3.877\%$.
Earthy matter—ash	$= 7.413\%$.
	100.000.

The usual formula is

$$h = 14500\,C + 62000\left(H - \frac{O}{8}\right); \quad \ldots\ldots\ldots (\textit{Dulong}),$$

in which the total heat-units, h, equal the heat-units due to the carbon, plus the heat-units due to the hydrogen less so much of the hydrogen as combines with the oxygen to form vapor of water.

As, in our example, the oxygen is not determined we shall put

$$h = 14500\,C + 62000\,H.$$

The heat-units due to the fixed carbon will be

$$0.8871 \times 14500 = 12863.$$

The volatile combustible, $C\,H_2$, consists of one-fourth hydrogen and three-fourths carbon. The heat-units due to its combustion will therefore be

$$\tfrac{3}{4} \times 0.03877 \times 14500 = 421.$$
$$\text{plus } \tfrac{1}{4} \times 0.03877 \times 62000 = 600.$$

Total heat-units $= 13884.$

13. *Example 2.*—Let it be required to determine the heat-units due to the combustion of a pound of bituminous coal, of which the composition is

Fixed Carbon, C	$= 60.301\%$.
Volatile combustible, $C\,H_2$	$= 30.839\%$.
Earthy matter—ash	$= 8.860\%$.
	100.000.

The heat-units due to the combustion of the fixed carbon will be

$$0.603 \times 14500 = 8744;$$

to the carbon of the volatile combustible, $\tfrac{3}{4} \times 0.3084 \times 14500 = 3355,$

and to the hydrogen of the volatile combustible, $\tfrac{1}{4} \times 0.3084 \times 62000 = 4780.$

Total, 16879.

The apparent value of the bituminous coal, as compared with the anthracite, is therefore,

$$\frac{16879}{13884} = 1.21+$$

14. *Composition of Air.*—Atmospheric air is a mechanical mixture of oxygen and nitrogen in the following proportions:

$$\left.\begin{array}{ll}\text{Oxygen,} & 23 \text{ parts} \\ \text{Nitrogen,} & 77 \;\;\text{“}\end{array}\right\} \text{by weight.}$$

or,

$$\left.\begin{array}{ll}\text{Oxygen,} & 21 \text{ parts} \\ \text{Nitrogen,} & 79 \;\;\text{“}\end{array}\right\} \text{by volume.}$$

For our present purpose, we may neglect decimals; also the impurities usually present, in very small quantities, in atmospheric air.

15. *Air Required to Burn 1 lb. of Hydrogen.*—One pound of hydrogen combines with eight pounds of oxygen, and forms nine pounds of water.

Each pound of air contains 0.23 of a pound of oxygen. Eight pounds of oxygen will therefore be supplied by

$$\frac{8}{0.23}=34.78 \text{ pounds of air.}$$

16. *Air Required to Burn 1 lb. of Carbon.*—If the combustion be perfect, the carbon will be converted into $C\,O_2$; the composition of which, by weight, is 6 parts carbon and 16 parts oxygen; or 1 part carbon and 2.67 parts oxygen.

To burn one pound of carbon, then, there will be required 2.67 pounds of oxygen; or

$$\frac{2.67}{0.23}=11.609 \text{ pounds of air.}$$

17. *Volume of Air Required to Support Combustion.*—One hundred cubic inches of dry air (bar. 30″ and ther. 60° Fahr.) weigh 31.0117 grains. The *volume* of one pound of air, under the same conditions will therefore be

$$\frac{7000}{17.28\times31.0117}=13.06 \text{ cubic feet.}$$

18. *Volume of Air, Modified by Temperature and Pressure.*—Air, and the other elastic fluids, under constant pressure, expand when heated as follows:

For each degree, Fahr.,

$\frac{1}{460}$ of their volume at 0° Fahr. $=0.00218$.

$\frac{1}{492}$ " " " " 32° " $=0.00203$.

$\frac{1}{520}$ " " " " 60° " $=0.00192$.

The expansion is uniform for equal increments of temperature.

19. *Absolute Zero.*—Absolute temperature, or the temperature above *absolute zero*, is the temperature by Fahrenheit's scale plus 460°.

Absolute zero is therefore 460° Fahr. below zero, or 492 below the freezing point of water.

At absolute zero a volume which is 460 at the zero of Fahren-

heit's scale, becomes *unity;* and since the volume varies, uniformly, with the temperature, it follows that the volume of a given weight of air, or other elastic fluid, will be proportional to its absolute temperature.

Thus, if t_a and t_a' be two absolute temperatures and t and t' be the corresponding temperature by Fahrenheit's scale, we have

$$t_a = t + 460^\circ.$$
$$t_a' = t' + 460^\circ.$$

If v and v' be the corresponding volumes, then will

$$v : v' :: t_a : t_a';$$

whence

$$v' = v \cdot \frac{t_a'}{t_a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

20. *Volume Affected by Pressure.*—The temperature being constant, the volume of a given weight of an elastic fluid will vary, inversely, as the pressure. In the case of air, the pressure is measured by the barometric column. If b and b' be two barometric pressures—in inches of mercury—corresponding to the two absolute temperatures t_a and t_a' respectively, or to v and v', Eq. (1) becomes

$$v' = v \cdot \frac{t_a'}{t_a} \times \frac{b}{b'}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In the case of air, let us take *one pound.* Thus, t being 60° and b, 30 inches, Eq. (2) becomes

$$v' = \frac{13 \times 30}{520} \times \frac{t_a'}{b'}$$
$$= 0.75 \frac{t_a'}{b'}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2^a)$$

Example.—Required the volume, in cubic feet, of one pound of air, at 90° Fahr., and barometer 28 inches.

$$t_a' = 90^\circ + 460^\circ = 550^\circ.$$

Then, Eq. (2^a)

$$v = 0.75 \times \frac{550}{28}$$
$$= 14.73 \; c.f.$$

21. *Weight of a Given Volume of Air, at Absolute Temperature t_a' and Barometer b'.*—

In Eq. (2^a) we have the volume, in cubic feet, of *one pound* of air.

Let V=the given volume, in cubic feet.
W=the weight of the same in pounds,

Then,

$$W=\frac{V}{v'}=\frac{V}{0.75\frac{t_a'}{b'}}$$

$$=1.333\ V.\frac{b'}{t_a'}.\ \text{lbs.} \qquad (3)$$

Example: Required the weight of 1000 cubic feet of air at 200° Fahr., and bar. 29 inches.

Here $t_a'=200°+460°=660°$.

Then, Eq. (3),

$$W=1.333\times 1000\times\frac{29}{660}$$
$$=58.586 \text{ pounds.}$$

22. *Volume of a Pound of any Elastic Fluid.*—Other things being the same, the volumes of equal weights of different elastic fluids are proportional, inversely, to their specific gravities.

The following specific gravities will be found to be convenient and useful:

Air	1.000.	Steam, 212°	0.488.
Nitrogen.	0.972.	Hydrogen.	0.070.
Carbonic Acid	1.524.	Oxygen	1.104.

Let δ = the specific gravity.

Then—dropping the primes—for the volume v, of one pound of any elastic fluid or gas, at absolute temperature t_a and pressure b inches of mercury we shall have, from Eq. (2^a),

$$v=0.75\frac{t_a}{b.\delta.}\ \text{cubic feet} \qquad (4)$$

We shall also have for the volume V, of w pounds of any gas,

$$V=0.75\frac{w.t_a}{b.\delta}\ \text{cubic feet} \qquad (4^a)$$

Example.—Required the volume of one pound of hydrogen, at the temperature of 80° Fahr., and at a pressure of 28 inches of mercury.

$$t_a = 460° + 80° = 540°.$$

Then, Eq. (4).

$$v = 0.75 \times \frac{540}{28 \times 0.07}$$
$$= 206. + \text{ cubic feet.}$$

23. *Weight of any Volume V of an Elastic Fluid.*—Since, for equal volumes and pressures, the weights of gases vary, directly, as their specific gravities, Eq. (3) is adapted to this case by multiplying the right-hand member by δ.

Then

$$W = V.\frac{b.\delta.}{0.75\, t_a}$$
$$= 1.333\ V.\frac{b.\delta.}{t_a} \text{ pounds} \ldots\ldots\ldots\ldots\ldots (5)$$

Example.—Required the weight W, of 1000 cubic feet of $C\,O_2$ at the temperature of 520° Fahr., and under a pressure of 30 inches of mercury.

Here, $t_a = 460° + 520 = 980°$.

Then, Eq. (5),

$$W = 1.333 \times 1000 \times \frac{30 \times 1.524}{980}$$
$$= 62.24 \text{ pounds.}$$

Example 2.—Required the volume V, of 200 pounds $C\,O_2$, at 100° Fahr., and under a pressure of 30 inches of mercury.

Here. $t_a = 460° + 100° = 560°$.

Then, Eq. (4^a),

$$V = 0.75 \times \frac{200 \times 560}{30 \times 1.524}$$
$$= 1837 + \text{ cubic feet.}$$

24. *Volume of Air, at 60° Fahr., Required for the Perfect Combustion of One Pound, Each, of Different Fuels.*—

a. Of anthracite coal, in which

Fixed carbon = 88.7%.
Volatile combustible = 3.877%.

The weight of air will be as follows:

For the fixed carbon, $0.887 \times 11.6 =$		10.289 lbs.
" " carbon of the vol. com.	$\frac{3 \times 0.03877}{4} \times 11.6 =$	.337 "
" " hydrogen,	$\frac{0.03877}{4} \times 34.78 =$	.337 "
Total............................		10.963 lbs.

The volume of this air, at 60° Fahr., will be

$$V = 10.693 \times 13 = 142.52 \text{ cubic feet.}$$

b. Of, semi-bituminous coal, in which,

Fixed carbon $= 73.78\%$.
Volatile combustible $= 14.62\%$.

In this case the weights of air will be as follows:

For the fixed carbon, 0.7378×11.6		$= 8.558$ lbs.
" " carbon of the vol. comb.	$\frac{3 \times 0.1462}{4} \times 11.6 =$	1.339 "
" " hydrogen,	$\frac{0.1462}{4} \times 34.78$	$= 1.339$ "
Total..............................		11.236 lbs.

The volume of this air, at 60° Fahr., will be

$$V = 11.136 \times 13 = 146.07 \text{ cubic feet.}$$

c. Of bituminous coal, in which,

Fixed carbon $= 60.31°$.
Volatile combustible $= 30.34°$.

Here, the weight of air required will be as follows:

For the fixed carbon, 0.6031×11.6		$= 6.996$ lbs.
" " carbon of the vol. comb.,	$\frac{3 \times 0.3034}{4} \times 11.6 =$	2.636 "
" " hydrogen,	$\frac{0.3034}{4} \times 34.78$	$= 2.636$ "
Total..............................		12.268 lbs.

The volume of this air, at 60° Fahr., will be

$$V = 12.268 \times 13 = 159.48 \text{ cubic feet.}$$

Collecting our results, we have,

For anthracite coal,	142.52	cubic feet.
" semi-bituminous coal,	146.07	" "
" " " "	159.48	" "
Sum,	448.07	" "
Mean, volume,	149.36	" "

We may therefore say that, in round numbers, 150 cubic feet of air, at 60° Fahr., will furnish oxygen enough to perfectly burn one pound of average coal.

In order to insure perfect combustion, however, it is generally necessary to admit more air than is chemically necessary to accomplish the desired end. The excess depends upon a variety of circumstances; such as thickness and condition of the fire, size and kind of coal used, rapidity of combustion, etc., etc.

In order to provide, liberally, for all contingencies, *double* the quantity of air, chemically necessary, or 300 cubic feet, should be admitted to the furnace for each pound of coal burned.

This is equivalent to

$$\frac{300}{13} = 23 + \text{pounds}$$

of air for each pound of coal.

One-half of this air supplies the oxygen needed for the combustion, while the other half mingles with the products of combustion, is heated by them and passes to and out of the chimney with them.

The products of combustion, obviously, consist of $C\,O_2$, N, vapor of water, air and, possibly, a small quantity of $C\,O$.

25. *Mean Specific Gravity of the Products of Combustion and of the Excess of Air.*—In order to determine the volume of the $C\,O_2$, as compared with that of its oxygen, let us take the $C\,O_2$ resulting from the perfect combustion of a pound of carbon, and put

W=weight of the $C\,O_2 = 1 + 2\frac{2}{3} = 3\frac{2}{3}$.
w= " " oxygen in same$=2\frac{2}{3}$.
v_c=volume of the $C\,O_2$.
v_o= " " its oxygen.
Δ=density of the $C\,O_2$.
δ= " " oxygen.
g=acceleration of gravity.

Then,

$$W = g\,\Delta\,v_c,$$

and

$$w = g\,\delta\,v_o;$$

whence

$$\frac{W}{w} = \frac{\Delta\,v_c}{\delta\,v_o}$$

and

$$v_c = v_o \cdot \frac{W\delta}{w\Delta}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Now, in place of Δ and δ we may write the specific gravities and CO_2 and O, respectively; (a) then becomes

$$v_c = v_o \times \frac{3\frac{2}{3} \times 1.104}{2\frac{2}{3} \times 1.524} = 0.996\,v_o:$$

from which it appears that the volume of CO_2 is very nearly the same as was that of its oxygen, before the combustion.

To determine the volumes of the products of combustion, and of the excess of air, in detail, take the first example of Art. 24, in which

Fixed carbon	=0.887 lb.
Carbon in $CH_2 = \frac{3}{4} \times 0.03877$	=0.029 "
Total............	0.916

and in which the hydrogen $= \frac{1}{4} \times 0.03877 = 0.00969$ lb.

The carbon combines with $2\frac{2}{3} \times 0.916 = 2.442$ pounds of oxygen, forming $2.442 + 0.916 = 3.358$ pounds of CO_2, and setting free $\frac{77 \times 2.442}{23} = 8.088$ pounds of nitrogen.

The hydrogen of the volatile combustible combines with $8 \times 0.00969 = 0.07752$ of a pound of oxygen, forming $0.00969 + 0.07752 = 0.08721$ of a pound of vapor of water, and setting free $\frac{77 \times 0.07752}{23} = 0.261$ of a pound of nitrogen.

Thus we have, altogether, from the perfect combustion of a pound of anthracite coal,

CO_2,..	3.358	lbs.
Vapor of water..................	0.0872	"
Nitrogen set free in the formation of CO_2.....8.088		"
" " " " " " vapor of water.0.261		"
Total nitrogen....................................	8.349	"
Surplus air $= 0.92 \times 11.6 + 0.338$*=................	11.010	"
Total weight of products=............	22.8042	"

*$0.261 + 0.07752 = 0.33852$ lbs. air.

The volumes of these several products, at temp. 60° Fahr. and bar. 30 inches, will be as follows:

$$\text{Vol. of } CO_2 = \frac{3.358 \times 13}{1.524} \qquad = \ 28.644 \text{ cu. ft.}$$

$$\text{Vol. of vapor of water} = \frac{0.0872 \times 13}{0.488} \qquad = \ 2.323 \quad \text{"}$$

$$\text{Vol. of nitrogen} = \frac{8.349 \times 13}{0.972} \qquad = 111.664 \quad \text{"}$$

$$\text{Vol. of surplus air} = 11.010 \times 13 \qquad = \underline{143.130} \quad \text{"}$$

Total volume.................. 285.761 "

Finally, multiplying the volumes by their respective specific gravities, and dividing the sum of the products by the sum of the volumes, we obtain the *mean specific gravity* of the mixture, as follows:

CO_2,	$28.644 \times 1.524 =$	43.654
Vapor of water,	$1.820 \times 0.488 =$	0.888
Nitrogen,	$111.664 \times 0.972 =$	108.537
Air,	$143.130 \times 1.000 =$	143.130
	285.761	296.209

Now $\qquad 285.761 \times \text{mean spec. grav.} = 296.209$

$$\therefore \text{mean specific gravity} = \frac{296.209}{285.761} = 1.04.$$

If we reject the excess of air, the mean specific gravity of the essential products of the combustion will be

$$\frac{296.209 - 143.130}{285.761 - 143.130} = 1.08.$$

26. *Temperature of the Furnace.*—In order to determine the temperature of the furnace in the case of perfect combustion, with normal supply of air,

Let

W = weight of the product of combustion, per pound of coal, in pounds.

c = specific heat of the products of combustion.

H = total number of heat-units due to the combustion of one pound of the coal.

T = temperature of the furnace.

Then, obviously,

$$W c\, T = H;$$

whence

$$T = \frac{H}{Wc} \quad \text{.......................} \quad (6)$$

Now the weight of the products of combustion, W, must be equal to the sum of the weights of the combustible elements of the fuel, plus the weight of the air admitted to the furnace. These are, in our example, (Art. 25), after the chemical changes.

CO_2,	3.358	lbs.
Vapor of water,	0.0872	"
Nitrogen,	8.349	"
$W =$	11.7942	"

For the specimen of coal, we have found, (Art. 12) $H = 13,884$.

The specific heat of the products of combustion may be taken as $c = 0.237$.

Substituting these several values in Eq. (6) we get

$$T = \frac{13884}{11.794 \times 0.237} = 4967^\circ \text{ Fahr.};$$

or, in round numbers, say 5000°.

If a double quantity of air be admitted to the furnace, the value of W will be increased by the weight of the excess of air; or, in this case

$$W = 11.794 + 11.010 = 22.804 \text{ pounds.}$$

Then, Eq. (6),

$$T = \frac{13884}{22.804 \times 0.237} = 2567^\circ + \text{ Fahr.};$$

or, in round numbers, say 2500°.

28. *Quantity of Heat Utilized.*—The heat of combustion, except that which passes to the chimney and produces draught—and except the variable, but relatively small, quantity lost by radiation—may be regarded as utilized in the boiler.

Neglecting, for the present, the loss of heat by radiation, and assuming that the total heat of combustion is expended upon the boiler and in producing draught, the two parts thus expended will evidently be proportional to the temperature of the

furnace, less the temperature in the chimney, to the temperature in the furnace, less the temperature of the external air.

Thus, if

T_f = temperature of the furnace,
T_c = " in the chimney,
and T_a = " of the external air,

the percentage of the heat of combustion which is expended upon the boiler, or the percentage of heat utilized, will be

$$H_u = 100 \frac{T_f - T_c}{T_f - T_a} \qquad (7)$$

In the case where only so much air as is chemically necessary for the combustion is admitted to the furnace, or where the temperature of the furnace is 5000°, and where the temperature in the chimney is 500°, we have—the external temperature being 60°,—

$$H_u = 100 \times \frac{5000 - 500}{5000 - 60} = 91.09\%.$$

If $T_f = 2500°$, or if a double supply of air be admitted to the furnace, we have

$$H_u = 100 \times \frac{2500 - 500}{2500 - 60} = 81.3\%.$$

In a similar manner we may determine the percentages of heat utilized when 1.2, 1.4, 1.6 and 1.8 times the air chemically necessary for the combustion are admitted to the furnace. The results are presented in the following table:

Air Admitted.	T_f	T_c	H_u
1	5000°	500°	91.09%.
1.2	4500°	500°	89.13%.
1.4	4000°	500°	87.17%.
1.6	3500°	500°	85.21%.
1.8	3000°	500°	83.25%.
2.	2500°	500°	81.30%.

It will be observed that the value of H_u is increased by diminishing the excess of air admitted into the furnace, and also by diminishing the temperature in the chimney. It clearly follows that, for a given fuel, the value of H_u will be a maximum when the excess of air is a minimum, and when the excess of the temperature in the chimney above the temperature of the boiler is also a minimum.

A high furnace temperature is insured by a proper arrangement of the furnace, and by skillful management of the fires.

A low temperature in the chimney is insured by providing ample boiler surface for the absorbtion of the heat of combustion. The minimum limit of chimney temperature is, of course, the temperature of the boiler, at the point where the products of combustion leave it, on their way to the chimney.

28. *Absolute Value of Heat Utilized.*—We may determine the absolute value of the heat utilized, in heat-units, under each set of conditions in the foregoing table, by multiplying the successive values of H_u by 13884.

We thus get, for

Air	1. ,	12647	units.
"	1.2,	12379	"
"	1.4,	12111	"
"	1.6,	11843	"
"	1.8,	11575	"
"	2.	11288	"

The loss by radiation, whatever it may be, must, of course, be deducted from the above values.

In a similar manner the number of heat-units which may be utilized from the combustion of a pound of any specimen of fuel may be approximately determined.

The importance of adopting every practical expedient for reducing the loss of heat by radiation, and of providing sufficient boiler surface to insure the absorbtion of the largest practicable percentage of the heat of combustion, needs to be kept constantly in view.

29. *Loss of Heat by Radiation.*—Experiments made for the purpose of determining the loss of heat by radiation, under any given set of conditions liable to be met with in practice, have been exceedingly rare. For this reason we are not able to estimate, with any degree of certainty, the loss suffered from this cause, in any practical case.

Fig. 1 represents the apparatus used by the writer, in the spring of 1878, for the purpose of determining the relative efficiencies of different modes of protecting steam boiler and pipe surfaces.

A is a cylindrical reservoir, of wrought iron, formed by capping the ends of a piece of seamless tube 30 inches in length,

$3\frac{1}{2}$ inches external diameter and $\frac{1}{4}$ inch thick. To the top of this was connected a half-inch pipe, *ss*, as shown, leading from the centre of a 5-inch steam pipe, which supplied steam to an engine which was constantly in operation during the progress of the experiments. *gg* is a glass water-gauge; *v*, a small cock, which was so adjusted as to admit the passage of water only; *BB*, a vessel kept full of ice or snow; and *c*, a tin vessel in which was collected the water of condensation in the reservoir *A*.

Steam was maintained at a nearly uniform pressure of 50 pounds above the atmosphere.

Three experiments were made, under each of four sets of conditions, and each experiment continued three hours.

The temperature of the air, about the reservoir, was maintained at about 70° Fahrenheit.

(*a*.) When the surface of the reservoir was unprotected, and when the temperature of the steam in the reservoir and of the external air were 300° and 70° respectively, the loss of heat measured by the condensation, was 1271.28 units per square foot of surface per hour; equivalent to the heat due to the combustion of about one-tenth of a pound of anthracite coal in the same time.

(*b*.) When the surface of the reservoir was covered with one inch of Riley Bros.' felting, with an air space of one inch, the other conditions being unchanged, the loss of heat was 825.83 units per square foot, per hour.

(*c*.) When one inch of the felting and one inch of "mineral wool" were used, the felting being outside, the loss of heat was 683.03 units, per square foot, per hour.

(*d*.) When the protection consisted of two inches of the felting, the other conditions being as before, the loss of heat was 751.06 units, per square foot of surface, per hour.

Thus it appeared, from the results of the experiments referred to, that one inch of "mineral wool," covered with one inch of the felting, constituted the most effective protection, and that it effected a saving of nearly one-half of the loss from the unprotected surface of the reservoir, under the conditions of the experiments.

II. Proportions of Boilers—Chimneys.

30. *Coal Consumption.*—The consumption of fuel under a steam boiler will depend upon the power to be maintained, and upon the work due to the combustion of one pound of coal per hour.

If W=the work to be done, in horse-power (H-P),
w= " " of one pound of coal, in horse-power,
C= " hourly coal consumption, in pounds.

Then

$$C = \frac{W}{w}. \quad \text{(8)}$$

The value of w varies from about 0.5 to about 0.2 of a horse power; very rarely exceeding the former value, and occasionally falling below the latter.

For convenience, however, we may classify engines into four classes, with reference to the values of w, as follows:

1. First-class modern condensing engines, $w = 0.50$
2. Second-class condensing engines, $w = 0.33$
3. Large, non-condensing engines, $w = 0.25$
4. Small, " " " $w = 0.20$

Among the first-class, may be ranked high-duty pumping and steam-ship engines; among the second-class, good pumping, and ordinary steam-ship engines; although in the latter, w sometimes exceeds 0.33

Example.—Required the hourly coal consumption of a second-class, or good pumping engine of 300 horse-power.

Eq. (8) gives.

$$C = \frac{300}{0.33} = 900 \text{ pounds.}$$

31. *The Grate Surface.*—This depends upon the power to be maintained, and upon the hourly rate of combustion, in pounds, per square foot of grate. With natural draught the hourly rate of combustion will vary from 5 to 12 pounds; while with artificial draught the rate is increased from 12 to perhaps 30 pounds, or more; while in the locomotive the rate sometimes reaches 140 lbs.

Slow combustion, when it is practicable, is found to be more economical, for a given boiler or heating surface, because the ratio of the heat produced, in a given time, to the surface provided for its absorption is smaller; thus insuring the absorption,

by the boiler, of a larger percentage of the heat, and allowing a smaller percentage to pass to the chimney.

Circumstances, however, in some cases preclude the practicability of providing sufficient grate surface to insure a moderate rate of combustion; as in the case of a locomotive, where the space is wanting, and in the case of river steamers, designed for freighting, where the space is too valuable, for stowing cargo.

In first and second-class engines, 8 to 10 pounds per hour, per square foot of grate, is an economical rate of combustion. When less than the normal power is required, the rate of combustion may be lessened, economically; and when more than the normal power is required, the rate may be increased; at the expense, however, for the time, of the normal measure of economy.

If G be the grate area, in square feet, and c be the hourly rate of coal consumption, in pounds, per square foot—C being the total hourly coal consumption, in pounds, as before—we shall have

$$G = \frac{C}{c}. \qquad (9)$$

Substituting the value of C, Eq. (8), we get

$$G = \frac{W}{w.c}. \qquad (9^a)$$

Example.—Required the grate surface, in square feet, necessary to maintain a power of 300 horses—the rate of coal consumption being 9 pounds per hour.

Eq. (9^a) gives

$$G = \frac{300}{0.33 \times 9} = 100 \text{ square feet.}$$

The length of the fire-grate—or its depth—is limited, for convenience in managing fires, to about 6 feet. It is sometimes greater than 6 feet, but is usually less; the exact length, or depth, depending upon circumstances.

In our example, if we make the length of the grate 6.0 feet, and the width of each grate 5.5 feet, there will be required $\frac{100}{6 \times 5.5} = 3$ furnaces.

Three cylindrical boilers, either 5 feet or 5.5 feet, in diameter, would be adapted to these furnaces.

32. *Area over the Bridge-wall.*—The bridge-wall is a wall at the rear of the furnace, connecting the side walls, extending

above the level of the grate and terminating at such a height as to leave a suitable space, between its top and the boiler, for the passage of the products of combustion from the furnace, backward, underneath the boiler. The extent of this space, between the top of the bridge-wall and the bottom of the boiler, is designated the "area over the bridge-wall." Its form, for cylindrical boilers set in brick-work should be as shown at *a*, Fig. 2, in which *A* is the ash-pit, *F* the furnace, and *F'* a space over the top of the boiler and included between it and the brick covering. With this arrangement the entire boiler is enveloped in the products of combustion, which leave it at the rear.

Careful experiment has shown that the area over the bridge-wall should be equal to *one-ninth* of the grate surface.

The arrangement shown in Fig. 2 may be regarded as standard, for return tube cylindrical boilers.

Other forms of boiler have an equivalent to the area here considered—although they may have no bridge-walls—in the area of the passage, or passages, from the furnace into the boiler-flues, or among its tubes.

We do not undertake to describe the various forms of steam boiler, as we are chiefly concerned with their general, and essential proportions, and not with their details.

33. *The Calorimeter.*—This, in the cylindrical boiler, is the aggregate sectional area of the return-tubes; and is the section of the stream of products of combustion which, when the boiler is in operation, passes through the boiler, from rear to front.

It is very important that the calorimeter, like the area over the bridge-wall, should sustain a proper relation to the grate surface. This relation, as determined by very many carefully conducted experiments, is as follows:

For all rates of combustion, the calorimeter, should be equivalent to $\frac{4}{31}$ of the area of the grate; it may be as great as $\frac{1}{7}$, but should never be less than $\frac{1}{8}$ of the grate.

In our example we have, in each furnace, 33 square feet of grate. The calorimeter should therefore be $\frac{33}{8} = 4\frac{1}{8}$ + square feet.

In order to provide for this area, let it be assumed that tubes, $3\frac{1}{2}$ inches in diameter will be used. The sectional area of each will be 9.621 square inches; and the required number of tubes,

$$\frac{4.125 \times 144}{9.621} = 62.$$

In order to accommodate this number of 3½ inch tubes, the boiler should be 5.5 feet, or 66 inches, in diameter.

In a battery of well-proportioned and well-set cylindrical boilers, each boiler being 5 feet in diameter, and having 58-3½ inch tubes, the calorimeter of each boiler is 3.875 square feet and its grate, $5.5 \times 5.5 = 30.25$ square feet. The ratio of calorimeter to grate area, in this case, is

$$\frac{3.975}{30.25} = \frac{1}{8} +$$

34. *The Heating Surface.*—This, in practice, varies from 25 to 50 times the grate surface. Careful experiments indicate that, for vertical, water-tubular boilers, the best ratio is 35 to 1; while for horizontal fire-tubular boilers, the best ratio is 45 to 1.

Our example is of the latter class. Therefore, Eq. (9^a), the heating surface of each boiler should be

$$H = 45\,G = \frac{45\,W}{w\,c}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

If $w = 0.33$, and $c = 9$ pounds, (10) reduces to

$$H = \frac{45}{0.33 \times 9} W = 15.15\,W. \quad \ldots\ldots\ldots\ldots (10^a)$$

In other words, there should be, in this case, about fifteen square feet of heating surface, per horse power. It does not follow, however, that every horizontal fire-tubular boiler having 1500 square feet of the heating surface is an economical or efficient 100 horse-power boiler. Other conditions must be satisfied, as well.

In our example, in which we are considering the general design of a battery of three boilers, of 100 horses each, the grate area of each boiler has been fixed at 33. square feet.

The heating surface should therefore be

$$33 \times 45 = 1485 \text{ square feet.}$$

The heating surface per lineal foot of a 66-inch boiler, having 62-3½-inch tubes, will be as follows:

Circumference	(5.5 ft. dia.), =	17.279 feet.
"	(3½ in. dia. × 62), =	56.811 "
Total heating surface per ft.,		74.090 sq. ft.

The length of the boiler will therefore be

$$\frac{1485}{74.09} = 20 \text{ feet,}$$

This length may seem to be excessive. It may be reduced to 18 feet without materially impairing its efficiency; or, by employing a larger number of smaller tubes, all of the proportions, and the full heating surface, may be secured in a length of 18 feet.

The heating surfaces at the ends of the boiler are treated as equivalent to so much of the surface of the cylindrical shell of the boiler as is not directly exposed to the products of combustion, and are therefore omitted in the calculation.

35. *Thickness of the Shell.*—The thickness of the shell of the boiler depends upon the pressure to which it is to be subjected, upon the diameter of the boiler, and upon the tensile strength of the material.

The best material, whether it be iron or low steel, is that which, of its kind, has a moderate tensile strength and great ductility.

Longitudinal seams should be double riveted, and circular seams, single riveted.

According to Fairbairn's experiments the two kinds of joint utilize the following percentages of the entire section of the material employed.

Single riveted joints, 56%.
Double " " 70%.

In order to construct a convenient formula for the determination of the proper thickness of the shell of a cylindrical boiler, let

p = working steam pressure, in pounds per square inch, above the atmosphere.
t = thickness of the metal in inches.
m = a coefficient, representing the fraction of the section of the material which is utilized by means of the riveting of the longitudinal seams.
d = diameter of the boiler in inches.
s = ultimate resistance of the material, in pounds per square inch of section.
f = safety factor.

Then, the effect of the working steam pressure, per linear inch of the boiler, along its axis, will be $p\,d$; while the bursting pressure will be measured by $f.\,p.\,d.$

This last pressure will be resisted by the ultimate strength of

the material of each linear inch of the boiler, which will be expressed by

$$2\ m.\ t.\ s.$$

Equating these expressions, we have

$$2\ m.\ t.\ s. = f.\ p.\ d.,$$

whence

$$t = \frac{f\ p\ d}{2\ m\ s}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

If $f = 5$, $m = 0.7$ and $s = 45000$, (11) becomes

$$t = 0.00008\ p\ d. \quad \ldots\ldots\ldots\ldots\ldots\ldots (11^a)$$

If, in our example, in which $d = 66$ inches, we make $p = 75$ pounds, (11^a) gives

$$t = 0.396 \text{ of an inch.}$$

Say, $\frac{3}{8}'' + \frac{1}{64}'' = \frac{25}{64}''$. In view of our liberal safety factor, $\frac{3}{8}$ of an inch may be regarded as ample. The heads should be $\frac{1}{2}''$ to $\frac{5}{8}''$ thick.

Figures 3, 4, 5 and 6 show the arrangement of the plates and heads, the manner in which the tube ends are secured in the heads, the longitudinal section of a boiler and its brick setting, where the products of combustion pass under, through and over the boiler, and a transverse section, through the furnace of an internally fired boiler—where the furnace is surrounded, except at the door, by water.

36. *Determination of the Temperature at the Base of the Chimney.*—The temperature of the products of combustion at the base of the chimney is determined by some form of pyrometer; or, approximately, by metals which fuse at proper, known, temperatures—or by fusible alloys.

Pyrometers are of various forms; some of which may be described as follows:

(*a*). Dr. Siemens' Pyrometer: This consists of a copper cup, surrounded by felt, and having a capacity of a pint. In connection with this are used a number of small copper balls, having such an aggregate capacity for heat that the heat given up by a reduction in their common temperature, of 50° Fahr., is sufficient to raise the temperature of a pint of water 1° Fahr.

Suppose these copper balls to be exposed, in the gases at the base of a chimney, until they are raised to the same temperature. Let the balls then be quickly transferred to the copper

cup, containing a pint of water at 60° Fahr., and suppose that the temperature of the water is thereby raised to 69°.

This will indicate that the balls were exposed to, and acquired, a temperature of

$$50\,(69^\circ - 60^\circ) + 69^\circ = 519^\circ.$$

(*b*). *Improvised Pyrometers.*—In the absence of a standard pyrometer, recourse may be had to an improvised arrangement which will give a fair approximation to the truth. Take, say, five pounds of small, and nearly uniform fragments of iron or copper, and expose them to the action of the gases at the base of the chimney; then place them in, say, ten pounds of water, at a known temperature, in a wooden vessel, and note the resulting temperature of the water.

With these data the required temperature may be ascertained as follows:

Let w = weight, in pounds, of the metal fragments.
w_1 = " " " water.
t_1 = its temperature.
t_2 = temperature of the water as raised by the heated metal.
h = the specific heat of the metal, and
t = the temperature to be ascertained.

Then, the number of heat-units contained in the metal, after its exposure and before it is placed in the water, will be expressed by

$$w \,.\, h \,.\, t.$$

while the number of heat-units contained in the water, at the same instant will be expressed by

$$w_1\, t_1\,.$$

After the metal has been placed in the water, and both it and the water have assumed the common temperature t_2, the total heat-units in both will be expressed by

$$w\, h\, t_2 + w_1\, t_2.$$

As no heat has been lost, we may equate this last equation with the sum of the two former expressions.

Then

$$w\, h\, t + w_1\, t_1 = (w\, h + w_1)\, t_2$$

whence

$$t = \frac{(w\, h + w_1)\, t_2 - w_1\, t_1}{w\,.\,h.} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

Dulong and Petit have found the following values for h, for wrought iron:

Between $32°$ and $212°$, h		$= 0.1098$
" $32°$ " $392°$, h		$= 0.1150$
" $32°$ " $572°$, h		$= 0.1218$
" $32°$ " $663°$, h		$= 0.1255$
Of white cast iron, h		$= 0.1298$
Copper, between $32°$ and $212°$, h		$= 0.094$
" " $32°$ " $572°$, h		$= 0.1013$

The specific heat of zinc is, practically, the same as that of copper.

Example.—If $w = 5$ lbs., $w_1 = 10$ lbs., $t_1 = 40°$, $t_2 = 65°$ and $h = 0.12$, *Eq.* (12) gives

$$t = \frac{(5 \times 0.12 + 10)65 - 10 \times 40}{5 \times 0.12}$$

$$= \frac{689 - 400}{0.6} = 482°.$$

The following are the melting points of metals, which may be used when no other means are at hand:

Tin............$446°$	Lead............$608°$
Bismuth.......$504°$	Zinc............$680°$

(*c*). *Air Pyrometer of Pouillet and Regnault.**—Fig. 7. This consists of a hollow platinum globe, from which extends a narrow tube, connecting with a larger glass tube, bent at the bottom into the form of the letter U.

The globe A, and the narrow tube $A\,B$, are filled with air; while the larger glass tube, $B\,F\,E$, is filled with mercury. The globe A is inserted in the space where the temperature is to be determined. As the air becomes heated, it expands, its elastic force is increased, and it forces the mercury column in $B\,C$ to fall, while the column $D\,E$ rises; the column of mercury $E\,H$ measuring the excess of the tension of the heated air, in A, over that of the atmosphere.

Let $V =$ the volume of the air in the globe A, at a known temperature and barometer, b inches.

$V_1 =$ the increased volume $B\,H$, due to the heating of this volume of air.

$t =$ temperature of the heated air.

$h = E\,H$, in inches, which is to be observed.

*From Weisbach's Mechanics of Engineering; Du Bois' Translation.

BC is protected by a wooden screen, and may be maintained at a uniform temperature by keeping it surrounded by boiling sulphuric ether or alcohol.

If e be the coefficient of expansion of air and γ be the original density of the air, the weight of the contained air will be $V\gamma$.

After the exposure and expansion of the air, and when its tension has been raised from b to $b+h$, we may conceive the whole volume of air to be made up of two volumes; a volume V_1 at the original temperature, and a volume V, at the temperature t.

The weight of the original air will be $V\gamma$. After the expansion, the weight of the air expelled from the globe, still maintained at its original temperature, will be

$$V_1\gamma \times \frac{b+h}{b};$$

while the weight of the air still remaining in the globe, at the temperature t, will be

$$\frac{b+h}{b} \times \frac{V}{1+et} \cdot \gamma.$$

Equating and factoring, we get

$$V \cdot \gamma = \left(\frac{V}{1+et+V_1}\right) \cdot \frac{b+h}{b} \cdot \gamma$$

whence,

$$\frac{bV}{b+h} = \frac{V}{1+et} + V_1;$$

which, solved for t gives

$$t = \frac{1}{e} \; \frac{V.h + V_1(b+h)}{V.b - V_1(b+h)}. \quad \ldots\ldots\ldots\ldots (13)$$

If the mercury be allowed to run out, through the cock A, until the columns are at the same level, h will become zero, and (13) will reduce to

$$t = \frac{1}{e} \cdot \frac{V_1}{V - V_1} \ldots\ldots\ldots\ldots\ldots\ldots (13^a)$$

This was Pouillet's method.

If, on the contrary, mercury be introduced, through A, until the mercury in BC rises to B, the volume of the air will still be V, and $V_1 = 0$. In that case (13) reduces to

$$t = \frac{1}{e} \cdot \frac{h}{b}.$$

This was Regnault's method, which seems to be the more simple and practicable.

A small correction ought to be made for the increased capacity of the platinum bulb, due to the expansion of its walls.

(*d*). Mr. Siemens has employed an electrical pyrometer, the action of which is due to the increased electrical resistance of platinum wire, due to an increase in its temperature, and which is said to give very trustworthy results.

The electrical resistances are measured by the decomposition of water in a differential voltameter, consisting of two similar voltameters, so arranged as to divide the current of a battery between two wires, one of a known resistance, and the other having its resistance increased by exposing it to the temperature to be ascertained.

37. *Height of the Chimney.*—The requisite height of the chimney will depend upon the area of a section of its flue, upon the volume of the products of combustion to be discharged by it, and upon the difference between the temperatures of these products and of the external air.

(*a*). The velocity of the ascending current of gases in a given chimney will depend upon the volume to be discharged, per second, and upon the area of a section of its flue.

Let t_a = the absolute temperature of the gases in the chimney.
δ = specific gravity of these gases.
k = volume of gases, in cubic feet, per pound of coal burned.
c = coal consumption, in pounds per second.
A = area of section of flue, in square feet.
b = barometer, in inches.
v = velocity of current, in feet per second.

Then

$$v = \frac{c\ k}{A}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

k represents the volume of w pounds of the products of combustion resulting from the combustion of one pound of coal; which, (4[a]) Art. 22, is expressed by

$$k = 0.75.\frac{w\ t_a}{b\ \delta}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

The volume of the gases resulting from the combustion of c pounds of coal, will then be $c.\ k$.

Multiplying (b) by c, and substituting the result for $c.\ k.$, in (a) we get, for the velocity,

$$v = 0.75\,\frac{c.\,w.\,t_a}{b.\,\delta.\,A}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Now, for a double air supply, w may be taken (Art. 24) as equivalent to, say, 23 pounds of air, plus 0.9 of a pound of combustible; or $w = 23.9$ pounds. We have also found, for the same conditions, (Art. 25), that $\delta = 1.04$.

If, finally, $t_a = 500° + 460° = 960°$, we have (14).

$$v = \frac{0.75 \times 23.9 \times 960}{1.04} \times \frac{c}{bA}$$

$$= 16546.\frac{c}{bA}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(14^a)$$

If $b = 30$ inches,

$$v = 551.53\,\frac{c}{A}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(14^b)$$

Example.—In our former example, we had a coal consumption of 900 pounds per hour, in 3 boilers. Suppose, now, that we provide a chimney of sufficient capacity for double this number of boilers; or for an ultimate coal consumption of 1800 pounds per hour.

Then

$$c = \frac{1800}{60 \times 60} = 0.5 \text{ pound.}$$

Suppose, now, that the chimney be circular in section, and 4 feet 4 inches in diameter. Then $A = 14.6$ square feet. These values, of c and A, in (14^b) give, finally, for the current of gases in the chimney flue

$$v = \frac{551.53 \times 0.5}{14\,6} = 18.89 \text{ feet per second.}$$

(b). *Head due to Velocity.*—In Fig. 8, H being the height of the chimney, in feet, above the level of the fire-grate, the draught will be due to the difference in weight, between a column of the chimney gases, and a column of the external air; the common height of the two columns being H, and their sections being each equal to A. Let t_a' be the absolute temperature of the external air, and t_a that of the heated gases.

Suppose that the two columns have the common absolute temperature t_a', that the temperature of the gases be then raised to the

absolute temperature t_a, and that the resulting expansion of the gases occurs without any mingling of the gases with the external air. The column of heated gases will then extend to a height h, above the top of the chimney.

Now when the chimney is performing its proper function, the velocity of the current of hot gases, passing up its flue, will be a function of the height h.

We shall evidently have

$$H : H + h :: t_a' : t_a;$$

whence

$$h = \frac{H(t_a - t_a')^*}{t_a'}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

h represents the difference in volume of the two columns, but not the difference in their weights; for the specific gravity of the gases is 1.04 times that of the external air.

The height of column which represents the difference in weight of the two columns, under the conditions stated, is therefore

$$h_1 = \frac{H(t_a - \delta t_a')}{t_a} \ldots\ldots\ldots\ldots\ldots\ldots (c)$$

Peclet's experiments have shown that the velocity of the current in the chimney is

$$v = \sqrt{2gh'}$$

in which h', the pressure head inducing the velocity, is

$$h' = \frac{h_1}{13 + \frac{0.012\,l}{m}} \ldots\ldots\ldots\ldots\ldots\ldots (d)$$

In (d), l is the length of the flue, from the grate to the top of the chimney, and

$$m = \frac{A}{\text{circ. of flue}}.$$

In order now to find an expression for the head h', substitute for h_1, in (d), its value in (c).

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*According to Peclet, the draught of a chimney is best when $\frac{t_a}{t_a'} = \frac{25}{12}$: or when h is nearly equal to H.

Then we have

$$h' = \frac{H\,(t_a - t_a')}{(13 + \frac{0.012\,l}{m})\,\delta\, t_a'}\,; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (e)$$

and

$$v^2 = 2\,g\,h'$$
$$= 2\,g\,H.\frac{t_a - t_a'}{(13 + \frac{0.012\,l}{m})\,\delta\, t_a'}\,;$$

whence, finally,

$$H = \frac{v^2\,(13 + \frac{0.012\,l}{m})\,t_a'}{2\,g\,(t_a - \delta\, t_a')}\,; \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which, it will be observed, v varies as $\sqrt{H}$

Example.—In our example we have found

$$v = 18.98\, f.\, s.$$

Let, also, $l = 160$ feet; $m = \frac{1}{4} \times 4'\,4'' = 1.1$ foot, nearly; $\delta = 1.04$; $t_a' = 90° + 460° = 550°$; $t_a = 500° + 460° = 960°$. Substituting these several values in (15), we get

$$H = \frac{18.89^2\,(13 + \frac{0.012 \times 160}{11}) \times 550}{64.32\,(960 - 1.04 \times 550)}.$$
$$= 112.\ \text{feet}.$$

Here, $h = 0.81\, H$, nearly.

The chimney of the Troy Water Works Pumping Station, at Lansingburgh, built in 1880, was designed for a grate surface equal to about 0.9 of that which we have used; while the rate of combustion assumed in the design of that chimney was the same—9 pounds per square foot of grate, per hour.

It follows, then, that the coal consumption, at Lansingburgh, is equal to 0.9 of the assumed consumption in our example. Thus, the volume of the products of combustion, and therefore the velocity of the flow in the chimney, at Lansingburgh, will be 0.9 of the volume and velocity in our example—A, t_a and t_a' being the same in both cases.

We have, then, two velocities, v and $0.9\,v$, and the height H, corresponding to v in our example, to find the height of H' of the chimney at Lansingburgh.

Remembering that v varies as $\sqrt{H}$, we may write

$$v : 0.9\,v :: \sqrt{H} : \sqrt{H'}$$

whence

$$H' = 0.81\,H,$$
$$= 0.81 \times 112 = 90.72\ \text{feet}.$$

The actual height of the Lansingburgh chimney is 100 feet, and the diameter of its circular flue is 4 feet 2 inches

The dimensions of the Lansingburgh chimney were, however, determined in different manner, which, while much more simple, gives equally reliable results.

From numerous elaborate and carefully conducted experiments authorized by the U. S. government, and directed by Chief Engineer B. F. Isherwood, U. S. N., the proportions of what may be termed a "standard chimney" have been determined.

The "standard chimney" is 60 feet high, above the level of the fire-grate, and has a flue whose sectional area is equal to the calorimeter area, Art. 33; the latter, of course, sustaining its proper relation to the area of the fire-grate. The rate of coal consumption to which the "standard chimney" is adapted, is 12 pounds per square foot of grate per hour.

It has already been made to appear that the velocity of a current in a chimney varies as $\sqrt{H}$. It is also evident that the *capacity* of a chimney whose height is H, is proportional to the area A, of its flue, and may be measured by

$$A. \sqrt{H}.$$

If, now, we desire an equivalent chimney having a height H', the area of its flue, A', will be determined as follows:

$$A' \sqrt{H'} = A \sqrt{H} \ldots\ldots\ldots\ldots (a)$$

whence

$$A' = A \sqrt{\frac{H}{H'}}. \ldots\ldots\ldots\ldots (16)$$

If the area A', of the flue of the equivalent chimney be assumed, its height H', will be

$$H' = H.\frac{A^2}{A'^2}. \ldots\ldots\ldots\ldots (17)$$

If H and A be found for the "standard chimney" for 100 horse-power, and it be desired to determine the elements of a chimney for N times this power, we may write

$$A' \sqrt{H'} = N A \sqrt{H}. \ldots\ldots\ldots\ldots (b)$$

Assume a height H', which shall be in harmony with the surroundings of the chimney. Then, solving (b) for A', we get

$$A' = N A \sqrt{\frac{H}{H'}}. \ldots\ldots\ldots\ldots (16^a)$$

If A' be assumed, we get

$$H' = N^2 . H . \frac{A^2}{A'^2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (17^a)$$

Example.—The grate area at Lansingburgh is $6 \times 5.5 \times 5.5 = 181.5$ square feet.

The proper calorimeter area for this grate-surface, for the standard rate of combustion—12 pounds of coal per hour, per square foot of grate—is $\frac{181.5}{8} = 22\ 69$, say 22.7 square feet. The rate of combustion adopted was 9 lbs.: or $\frac{3}{4}$ of the standard rate. The calorimeter should therefore be only $\frac{3}{4}$ of the standard calorimeter; or

$$A = \tfrac{3}{4} \times 22.7 = 17 + \text{ square feet.}$$

This reduced standard chimney, then, has $A = 17.$ square feet, and $H = 60$ feet.

It was decided that the height of the chimney, H', should be 100 feet, and (16) gave for the area of the flue

$$A' = A\sqrt{\frac{H}{H'}}.$$

$$= 17 \times \sqrt{\frac{60}{100}} = 17 \times 0.77 = 13.09 \text{ sq. ft.}$$

13.09 square feet is the area of the circle whose diameter is about 4 feet 1 inch.

The flue of the Lansingburgh chimney is circular, and has an internal diameter of 4 feet 2 inches.

Example 2.—In the case where there were 6 furnaces, $6' \times 5.'5$ each, the total grate area was $6 \times 6 \times 5.5 = 198.$ sq. ft.

The assumed rate of combustion being, in that case, 9 pounds, the flue of the reduced standard chimney will be

$$A = \tfrac{3}{4} \times \tfrac{1}{8} \times 198 = 18.56 \text{ square feet, and } H = 60 \text{ feet.}$$

The flue was assumed to be circular, and to have a sectional area, $A' = 14.6$ square feet.

These data, in (17) give

$$H' = 60 \times \frac{18.56^2}{14.6^2} = 97. \text{ feet.}$$

This is 15 feet less than the height given by (15). The capacity of the chimney given by (15) is, therefore,

$$100\left(\sqrt{\frac{112}{97}}-1\right)=7.45\%$$

greater than that found by the method deduced from the U. S. government experiments.

The effect of this greater capacity is, of course, to create a stronger draught, to burn more coal and to impair the efficiency of the boilers by conveying away the products of combustion too rapidly.

The temperature of the gases in the chimney of larger capacity, 112 feet high was assumed to be 500° Fahr. It follows, therefore, that the chimney of smaller capacity, 97 feet high, and having a flue of the same diameter and sectional area, will produce a more moderate draught, will cause less coal to be burned, and will be more efficient; because it will afford the products of combustion more time in which to give up their heat to the boilers.

On this account the temperature of the gases in the chimney should be reduced considerably below 500°.

Fig. 9. shows the arrangement of the Lansingburgh chimney, and its connection with the main underground flue leading from the boilers. *F* is the underground flue, leading from the boilers, and having a sectional area slightly greater than the area of the chimney flue; *B B* is the outer brick shell, and *L L* the fire-brick lining, extending from the bottom of the flue to the top of the chimney. The fire-brick lining is 12 inches thick, up to a plane 40 feet above the grates, 8 inches thick for the next 40 feet, and 4 inches thick for the last 20 feet.

The outer shell, of common brick, faced with Croton brick, is 16 inches thick to 40 feet above the grates, 12 inches thick for the next 40 feet, and 8 inches thick for the last 20 feet.

Between the outer shell and the lining is an air space of variable thickness.

The foundation of the chimney is 18 feet square and extending about 18 feet below the surface, where it rests upon cemented clay and gravel, overlying solid rock.

The chimney is surmounted by a cast iron cap, cast in four pieces, which are bolted together, and which are provided with flanges which project downward about 3 inches, both on the outside and within the fire-brick lining.

At *D*, is a passage leading to the bottom of the flue, which is closed by an iron door, faced on the inside with fire-brick. The fire-brick lining terminates about one inch below the top of the outer shell, which supports the cast-iron cap, thus permitting free expansion of the lining without disturbance to the outer shell of the chimney.

In this plant, the products of combustion pass, from the furnaces, backward under the boilers, forward through the tubes, again backward, over the boilers, and then descend, vertically, about 15 feet, to an underground flue leading to the chimney.

Dampers are provided, for each boiler, which are placed at the points where the products of combustion finally leave the boiler surfaces. Dampers are also placed in the underground flue, leading from each of the two batteries of boilers, where they connect with the main flue leading to the chimney. This chimney is of ample capacity, and has performed its functions in an entirely satisfactory manner.

III. Properties of Water and Steam.

38. *Ice; Total Heat of Fusion.*—The specific heats of water, in its three forms, are:

Specific heat of ice = 0.504.
" " " water = 1.000.
" " " steam = 0.622.* (gaseous steam.)

The heat required to melt ice, and to convert the resulting water into steam, is made up of the sensible heat required to raise the temperature of the ice to the melting point, the latent heat of fusion—or the heat expended in melting the ice without raising its temperature—the sensible heat required to raise the temperature of the water from the freezing point to the temperature of evaporation, and the latent heat of vaporization.

M. Person's empirical formula for the latent heat of *fusion*, of non-metalic solids, is

$$l = (c' - c) \times (t + 256^\circ); \quad \ldots\ldots\ldots\ldots\ldots (19)$$

in which

c = the specific heat in a solid state.
c' = " " " " liquid "
t = " temperature of fusion—melting point, Fahr.
l = latent heat of fusion, per pound, in English units.

Example.—Required the number of heat-units which must be expended in melting one pound of ice, at 32°.

According to (19) this is

$$\begin{aligned} l &= (1 - 0.504) \times (32 + 256) \\ &= 0.496 \times 288 \\ &= 142.8 \text{ units.} \end{aligned}$$

39. *Water; its Weight at Different Temperatures.*—

If W = the weight of a cubic foot of water at the temperature of maximum density—39.1°, Fahr.
W' = the weight of a cubic foot of water at any other temperature, t.
and t_a = the absolute temperature = t + 460° Fahr.

Then, according to Rankine's empirical formula,

$$W' = \frac{2W}{\frac{t_a}{500} + \frac{500}{t_a}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

*Air, at 32° Fahr., being unity.

Example.—If $t = 100°$ and $t_a = 100° + 460° = 560°$.

$$W' = \frac{2 \times 62.425}{\frac{560}{500} + \frac{500}{560}} = 62.022 \text{ pounds.}$$

The following table, giving the weights of a cubic foot of water at temperatures ranging from the freezing point to 390°, will be found convenient.

Temperatures.	Weights of a cubic foot.	Remarks.
32°	62.418 lbs.	
35°	62.422 "	
39.1°	62.425 "	Maximum density.
40°	62.425 "	
46°	62.418 "	
50°	62.409 "	
52.3°	62.400 "	Convenient for calculation.
60°	62.372 "	
70°	62.313 "	
80°	62.232 "	
90°	62.133 "	
100°	62.022 "	
110°	61.868 "	
120°	61.715 "	
130°	61.563 "	
140°	61.381 "	
150°	61.201 "	
160°	60.991 "	
170°	60.783 "	
180°	60.548 "	
185°	60.430 "	
190°	60.315 "	
200°	60.081 "	
210°	59.82 "	
212°	59.64 "	
250°	58.75 "	
298°	57.27 "	Steam at 50 lbs. above atmosphere.
338°	56.14 "	" 100 " "
366°	55.29 "	" 150 " "
390°	54.54 "	" 205 " "

For intermediate temperatures the weights may be found by interpolation; or, better, by a curve of weights, constructed graphically, as shown in Fig. 10; in which the temperatures are laid off as abscissas, and the excesses of the weights above some standard weight—as 60 pounds—as ordinates.

For this purpose profile paper may be conveniently used; and such scales, both of temperature and weight, adopted as will permit the temperatures and weights to be read, to any desired degree of accuracy, by inspection.

About $\frac{1}{32}$, or 3.1%, of the weight of mean sea water, is made up of salt.

A saturated solution of salt contains about $\frac{12}{32}$, or 37.2% of salt and 62.8% of water. These were the proportions formerly used by U. S. naval engineers. Late authorities, however, place the salt in a saturated solution at 30%, and the boiling point at 227° Fahr.

40. *The Boiling Point of Water.*—The boiling point of water is the temperature at which tension of the vesicles of steam, formed by the action of the heat, becomes equal to the pressure upon the surface of the water. At this temperature the steam rises through the water and is liberated at its surface. The agitation of the water, caused by steam rising through it, and which varies with the quantity of heat applied, and the quantity of steam formed, is termed boiling.

The temperature of the boiling point increases with the pressure. It has been determined for various pressures, experimentally, and may be determined, approximately, by empirical formulæ which have been constructed for that purpose. With the tabulated values, to be given presently, we shall have no occasion to use these formulæ, and they are, therefore, omitted.

The following table contains, for each indicated pressure in pounds per square inch above zero, the corresponding values, of the pressure in inches of mercury, of the total heat above 32° in the water, of the total heat above 32° in the steam, and the specific volume of the steam.

The total heats are for one pound of water or steam in each case.

The specific volumes are the volumes of the steam compared with the water from which it is formed, at 39.1°, or at the temperature of maximum density.

Pressures.		Temperatures.	Total Heat—Above 32°.		Volumes;
Pounds.	Inches of Mercury.	Fahr.	In. 1 lb. Water.	In. 1 lb. Steam.	Water, at 39.1°, being Unity.
1.	2.037	101.36	69.431	1112.85	17982.20
5.	10.186	162.51	130.891	1131.51	4565.60
10.	20.372	193.20	161.870	1140.87	2373.08
14.6857	29.922	212.00	180.900	1146.60	1641.42
20.	40.744	227.95	197.079	1151.46	1219.73
30.	61.117	250.26	219.766	1158.27	826.33
40.	81.489	267.17	237.008	1163.43	627.90
50.	101.861	280.89	251.028	1167.61	508.29
60.	122.233	292.58	262.997	1171.18	428.31
70.	142.606	302.77	273.449	1174.28	371.08
80.	162.987	311.86	282.784	1177.06	328.08
90.	183.350	320.10	291.260	1179.57	294.60
100.	203.722	327.63	299.015	1181.87	267.81
110.	224.094	334.59	306.192	1183.99	245.86
120.	224.467	341.06	312.871	1185.96	227.55
130.	264.839	347.11	319.124	1187.81	212.00
140.	285.211	352.85	325.062	1189.56	198.78
150.	305.583	358.43	330.758	1191.26	187.26

The temperatures, total heats and volumes of steam corresponding to intermediate pressures, may be determined by interpolation, or by curves, constructed graphically, as suggested in Art. 39, for determining the weight of a cubic foot of water at any temperature not found in the table.

41. *Latent Heat of Steam.*—The latent heat of steam, or the latent heat of vaporization of water, may be calculated, directly, by an empirical formula deduced by Regnault from the results of his experiments.

If t be the temperature at which the evaporation takes place, and h be the number of latent heat-units per pound of steam, then

$$h = 966 - 0.7\,(t - 212°). \qquad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

If $t = 212°$, $h = 966$ units. (Tabular values, $1146.6 - 180.9 = 965.7$.)

It appears from (20) that the latent heat diminishes as the temperature—and therefore as the pressure—increases.

If $t = 248°$,

$$h = 966 - 0.7\ (248 - 212)$$
$$= 966 - 25\ 2$$
$$= 940.8 \text{ units per pound.}$$

Example.—Required the latent heat of vaporization of one pound of water, under the absolute pressure of 100 pounds per square inch (85.3 lbs. above the atmosphere),

Here $t = 327°.63$; therefore (20)

$$h = 966 - 0.7\ (327.63 - 212)$$
$$= 966 - 80.94$$
$$= 885.06 \text{ units.}$$

The tabular values gives

$$h = 1181.87 - 299.015$$
$$= 882.855 \text{ units.}$$

An exact agreement between the experimental values and those given by the empirical formula is not, of coure, to be expected.

42. *Total Heat of Evaporation.*—This is made up of the sensible heat of the water or steam—indicated by the thermometer—and the latent heat of evaporation. Regnault found that the total heat of evaporation—above 32°—increased, uniformly, with the temperature at which the evaporation took place. His empirical formula, for the total heat, above 32°, is

$$H = 1092 + 0.3\ (t - 32°); \quad \dots\dots\dots\dots\dots (21)$$

in which t is the temperature of evaporization and H the number of heat units, above 32°, in one pound of steam.

(21) may be reduced to the following form:

$$H = 1146 - 0.3\ (t - 212°). \quad \dots\dots\dots\dots (21^a)$$

If, in (21), $t = 32°$, $H = 1092$ units.

If, in (21^a), $t = 212°$, $H = 1146$ units—(1146.6 in table).

Adding 32 units, we get, for the total number of heat units above zero, 1146 + 32 = 1178.

If $t = 255°$, (21^a) gives

$$H = 1146 + 0.3\ (255 - 212)$$
$$= 1146 + 12.9 \text{ units.}$$
$$= 1158.9 \text{ units.}$$

Adding 32, we get for the total heat above zero, 1158.9 + 32 = 1190.9 units.

43. *Heat Required to Evaporate a Pound of Water from any Temperature t, and at any Temperature t'.*—

It is often necessary, for the purpose of comparing evaporations, where they are performed at different temperatures of feed-water, to reduce one evaporation to what would have been under the conditions of the other; or, to reduce both evaporations to what they would have been *from* and *at* 212°.

The heat required is equal to the difference between the total heats of the steam and of the feed-water. If we represent this heat by H_1 we shall evidently have, (21),

$$\left.\begin{aligned} H_1 &= 1092 + 0.3\,(t' - 32) - (t - 32 \\ &= 1114.4 + 0.3\,t' - t. \end{aligned}\right\} \quad \ldots\ldots\ldots (22)$$

If the temperature of the feed-water (t) be 200°, and that of the evaporation (t') 305°, we get

$$\begin{aligned} H_1 &= 1114.4 + 0.3 \times 305 - 200 \\ &= 1005.9 \text{ units.} \end{aligned}$$

It will be observed that H_1 diminishes as t increases; and it necessarily follows, that the number of pounds of steam made, per unit of fuel, will be greater, as t is greater. Hence the desirability, where it is practicable to do so, of employing heat which would otherwise be wasted, in heating the feed-water.

In order to reduce the evaporation from the temperature t and at the temperature t', to what it would have been from and at 212°,

Let $H_1' =$ heat units given by (22).
$H_1 =$ " " at 212°.
$w_1 =$ the evaporation, in lbs., from t and at t'.
$w =$ " " " " from and at 212°.

Then, since the weights of water evaporated will be inversely proportional to the numbers of heat-units per pound in each case, we may write

$$w : w_1 :: H_1' : H_1.$$

Whence

$$w = w_1 \frac{H_1'}{H_1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Example.—8 pounds of water were evaporated from $t = 104°$ and at $t' = 230°$, per pound of coal. Required the equivalent evaporation from and at 212°.

(22) gives us

$$H_1' = 1114.4 + 0.3 \times 230 - 104$$
$$= 1114.4 + 69 - 104$$
$$= 1079.4 \text{ units.}$$

The same formula gives us

$$H_1 = 1114.4 + 0.3 \times 212 - 212$$
$$= 966 \text{ units.}$$

Then, since $w_1 = 8$ pounds, we have (23)

$$w = 8 \times \tfrac{1079.4}{966}.$$
$$= 8 \times 1.117 = 8.936 \text{ pounds.}$$

The value of the ratio 1.117, of the total heats of evaporation, may be found directly from the temperatures, as follows:

(22) may take the form

$$H_1 = 966 + 0.3\,(t' - 212) - (t - 212°) \ldots\ldots\ldots\ldots (a)$$

If $t = 212°$ and $t' = 212°$, (a) becomes

$$H = 966.$$

Dividing (a) by this last value we get

$$\text{ratio, } \frac{H_1'}{H_1} = 1 + \frac{0.3\,(t' - 212) + (212 - t)}{966}$$

$$= 1 + \frac{148.4 + 0.3\,t' - t}{966} \ldots\ldots\ldots\ldots\ldots (24)$$

Taking the same example as before, in which $t' = 230°$ and $t = 104°$, we get from (24),

$$\text{ratio} = 1 + \frac{1.484 + 0.3 \times 230 - 104}{966}$$

$$= 1 + \frac{148.4 + 69 - 104}{966}$$

$$= 1 + \frac{113.4}{866} = 1 + 0.117 = 1.117,$$

as before.

44. *Vapor of Water and Gaseous Steam.*—The distinction between these conditions seems to be this: Vapor of water is the steam as it is formed, and escapes, from the water; it is steam in its normal, or saturated, condition; and it is usually designated "saturated" steam.

Gaseous steam is the same steam after its contained moisture

has been evaporated by the application of additional heat; it is usually designated " superheated " steam.

Superheating may, of course, be carried beyond the point at which saturated steam becomes gaseous steam.

The total heat of gasification, under constant pressure,* is

$$H_2 = a + c' (t' - t); \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which a is a constant, equal to the total heat of steam at 32° Fahr. = 1092 units; c' = specific heat of gaseous steam, at constant pressure = 0 475 ; t' = temperature of the gas ; t = the original temperature of the water, and H_2 = the number of heat-units required.

Example.—How many heat-units are required, in order to convert a pound of water at 32° into steam-gas at 212°.

Here,

$$H_2 = 1092 + 0.475 (212 - 32)$$
$$= 1177 \text{ units.}$$

Now, we have seen that, in order to convert the same water into saturated steam, only 1146 units of heat were required. Thus, under the conditions of our example, 1177 – 1146 = 31 units, or $\frac{100 \times 31}{1146}$ = 2.8 per cent., more heat is required to form gaseous steam, than is required to form saturated steam.

45. *Condensation Due to Work Performed.*— Saturated steam, when performing work, suffers a loss of heat, and, consequently condensation. The quantity of heat lost, and the resulting condensation are proportional to, and measured by, the heat equivalent of the work performed, plus the loss of heat by radiation.

This condensation may be observed in a glass cylinder, in which, after the cut-off, or after the admission of the steam to the cylinder is suppressed, and while expansion is taking place, the steam space, which was before perfectly clear, becomes filled with a dense mist.

Condensation in the cylinder may be prevented by superheating, or by adding to the saturated steam heat equivalent to the work to be done by it and to the loss by radiation.

Superheating has been resorted to in some cases; but whether with advantage or not is not quite clear. It is certain, however, that certain disadvantages attended its use; as, for example, the

*The specific heat of air, under constant pressure, is 1.41 times as great as its specific heat under constant volume.

destructive effect of the excessive heat upon the steam packings of the piston-rods and valve-stems.

When the work of a pound of steam is known, the effect of the addition of its equivalent, in heat, to the saturated steam, upon its temperature, can be readily estimated.

The addition of a heat-unit to a pound of saturated steam will raise its temperature

$$\frac{1}{0.475} = 2.105 \text{ degrees.}$$

If n be the number of heat-units represented by the work of a pound of steam, the requisite amount of superheating will be

$$2.105\ n. \text{ degrees.}$$

45½. *Quality of Steam.*—The quality of the steam which is supplied in any case, may be ascertained by measuring its heat, and comparing the result with the heat-units in standard, normal, or saturated steam. The apparatus for this purpose consists, essentially, of a vessel containing a known weight of water at a temperature somewhat below that of the surrounding air. Place this vessel as near as practicable to the main steam pipe, with which a connection should be made, and a small, well-protected pipe, or rubber hose, led to the vessel. This connection with the steam-pipe should project into the interior of the pipe, as far as its axis, and should there bend toward the boiler.

When all is in readiness, the vessel of water resting on the platform of a weighing scale, open the valve in the small pipe, and, after being satisfied that all water of condensation has escaped, turn the discharge of steam into the water until the weight of the water has been increased by an amount previously determined upon.

The vessel now contains its original water, plus the water resulting from the condensation of the added steam.

The temperature of the contents of the vessel has been raised, and the heat-units, originally in the water, have been increased, by the heat-units brought over in the steam which has been added.

If the added steam brought over the number of heat-units due to so much saturated steam, its efficiency is said to be unity; if it brought over less than the normal quantity of heat, its efficiency is said to be less than unity; while if more heat is

brought over than is due to saturated steam, the efficiency of the steam is greater than unity, and the steam is superheated.

If $H_1' =$ number of heat-units brought over by the added steam,

$H' =$ number of heat-units in same weight of saturated steam,

$E =$ efficiency of the steam,

$H =$ number of heat-units in a unit of steam,

we have

$$E = \frac{H_1'}{H'}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

In order to determine H_1', and the weight of water brought over, mechanically suspended, in the steam, let

$w =$ the weight of the water used, in pounds,

$t =$ its original temperature,

$w_1 =$ the weight of steam added,

$w' =$ the weight of the water brought over by the steam,

$w_2 = w_1 + w'. =$ total added weight,

$H =$ total heat of the steam, per pound, above *zero*,

$t_1 =$ its temperature,

$t_2 =$ final temperature of the water in the vessel.

Then, in order to determine the efficiency, we may treat w' as zero, and consider the entire added weight, w_1, as so much steam.

The heat given up, by the added steam, to the water in the vessel, is

$$H_2 = w(t_2 - t);$$

while the total heat due to w_1 pounds of steam above t_2 is

$$H' = w_1(H - t_2).$$

Therefore (26)

$$E = \frac{w(t_2 - t)^*}{w_1(H - t_2)}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Or, more precisely,

$$E = \frac{H_1'}{H'} = \frac{(w + w')t_1 - w\,t}{w_1\,H}. \qquad \ldots\ldots\ldots\ldots (27^a)$$

H is to be taken from the table, Art. 40; or it may be calculated by (21), Art. 42. In either case, 32 must be added to the result.

As already stated, if

$E = 1$, the steam is saturated.

$E < 1$, " " " wet.

$E > 1$, " " " superheated.

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*Approximately.

A formula for the weight of water, w', brought over in the steam—and resulting from condensation in the pipe—may be constructed as follows:

$$w_2 = w_1 + w',$$

or,

$$w_1 = w_2 - w'.$$

Then, the heat given up by w_1, to the water in the vessel, will be

$$w_1(H - t_2) = (w_2 - w') \,.\, (H - t_2);$$

and by w',

$$w'(t_1 - t_2);$$

or by both,

$$w_2(H - t_2) - w'(H - t_2) + w'(t_1 - t_2). \quad \ldots\ldots\ldots (a)$$

The heat received by the water in the vessel will be

$$w(t_2 - t). \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

Equating (a) and (b) we get

$$w'[(H - t_2) - (t_1 - t_2)] = w_2(H - t_2) - w(t_2 - t);$$

or,

$$w'(H - t_1) = w_2(H - t_2) - w(t_2 - t);$$

whence

$$w' = \frac{w_2(H - t_2) - w(t_2 - t)}{H - t_1}. \quad \ldots\ldots\ldots\ldots (28)$$

Example.—From a boiler, carrying 75 pounds of steam, 15.75 pounds of steam were delivered into a wooden vessel, containing 100 pounds of water at an initial temperature of 42°, raising the temperature of the water to 160°.

In this case, we have the following data: $t_1 = 320°$; $H = 1179 + 32 = 1211$; $w = 100$ lbs.; $w_1 = 15.75$ pounds; $t_2 = 160$; $w = 42°$.

Then, substituting in (27), we get

$$E = \frac{100\ (160 - 42)}{15.75\ (1211 - 160)} = 0.713.$$

Eq. (27^a) gives

$$E = \frac{(100 + 15.75) \times 160 - 100 \times 42}{15.75 \times 1179} = 0.751$$

Substituting in (28), and making $w_2 = 15.75$ we get

$$w' = \frac{15.75\ (1211 - 160) - 100\ (160 - 42)}{1211 - 320}$$

$$= 5.33 \text{ pounds.}$$

As this experiment occupied "over an hour," it was clearly vitiated by the condensation in the pipe; which must have fur-

nished a very large percentage of the 5.33 pounds of water which was delivered with the steam.

On April 8th, 1882, the writer repeated the tests of the steam furnished by this boiler, in the following manner:

In a galvanized iron pail, weighing 4 lbs. 1 oz, 10 pounds of water, at 62°, was placed. Steam was then admitted, during a period of $1\frac{1}{4}$ minutes, when the temperature of the water was 165°, and the weight had been increased by *one pound.* The pressure of the steam was 75 pounds, above the atmosphere, at which $t_1 = 320$.

Here the heat received by the contents of the pail was $10 \times (165 - 62)$	$= 1030$	units.
and by the pail itself, say $4.06 \times 0.12 \times 100$	$= 48.72$	"
Total..........................	1078.72	"

The heat due to a pound of steam, between the temperatures 165° and 320° = 1211 − 165 = 1046 units.

We then have

$$E = \frac{1078.72}{1046} = 1.03 - ;$$

which would indicate superheating, to the extent of 1078.72 − 1046 = 32.72 units to the pound; or about $\frac{32.72}{0.475} = 69 -$ degrees. It is not likely, however, that the iron pail received 48.72 units of heat as has been assumed—as its capacity was not more than half filled, and as the time was too short for the entire mass of the pail to attain the temperature of the water in its bottom.

If we disregard the pail and substitute the other data, in (27^a), we get

$$E = \frac{(10 + 1) \times 165 - 10 \times 62}{1 \times 1211} = 0.987.$$

Making $w_2 = 1$ lb. and substituting in (28) we get for the water delivered with the steam.

$$w' = \frac{1\ (1211 - 165) - 10\ (165 - 62)}{1211 - 320} = 0.018 \text{ lb.}$$

It is not unusual to have 5% of water delivered with the steam. Occasionally, a much larger percentage is delivered; especially when the steam space above the water, in the boiler, is too small, as compared with the draught upon it. In that case, the effect of the steam upon the body of water, as it rises rapidly

through it, resembles, somewhat the effect of the rapid escape of the gas from champagne when the cork is removed from the bottle which confines it.

This is illustrated in the results of a second test of the steam of the boiler referred to.

To 10 pounds of water, at 47°, was added 1.219 lb. of steam; when the resulting temperature was found to be 165°. The boiler pressure was falling, rapidly, and, at the conclusion of the test, was only 70 pounds; or $t_1 = 316°$.

With these data, in (27), we get

$$E = \frac{10\,(165 - 47)}{1.219\ (1211 - 316)} = 0.902$$

making $w_2 = 1.219$, and substituting in (28), we get

$$w' = \frac{1.219\,(1211 - 165) - 10\,(165 - 47)}{1211 - 316} = 0.143 \text{ lb.}$$

At the time of this test, owing to the rapid reduction in pressure, the temperature of the water, in the boiler, was greater than that due to the diminished pressure; the result was the rapid liberation of steam with water in suspension.

Enough has been said to indicate the great importance of employing accurate instruments, and of performing the test quickly, and with great care.

46. *Theoretical Evaporation per Unit of Fuel.*—It is shown, in Art. 28, that, with a double supply of air, and with a chimney temperature of 500°, 11288 units of heat may be utilized in the boiler, from the combustion of each pound of anthracite coal. Now, if steam be carried at a pressure of 90 pounds above zero, or say 75 pounds above the atmosphere, and if feed-water be supplied at 100° Fahr., the total heat of evaporation will be $1179.57 + 32 - 100 = 1111.57$ units; say 1112 units. The theoretical evaporation of a pound of average anthracite coal will therefore be

$$\frac{11288}{1112} = 10.15 \text{ pounds.}$$

With only so much air as is chemically necessary, 12647 units of heat may be utilized in the boiler from each pound of coal. In that case, the theoretical evaporation will therefore be

$$\frac{12647}{1112} = 11.37 \text{ pounds of water—or steam.}$$

The evaporation from and at 212°, at which the total heat of evaporation is 966 units, will be, where there is a double supply of air,

$$\frac{11288}{966} = 11.68 \text{ pounds.}$$

Where there is no excess of air, and where the combustion is practically perfect, the evaporation, per pound of coal, will be

$$\frac{12647}{966} = 13.09 \text{ pounds.}$$

47. *Actual Evaporation.* — In practice the evaporation, per pound of coal, ranges from 7 to 11 pounds; a fair average rate of evaporation is, perhaps, 8½ pounds of water per pound of coal.

When boilers are properly proportioned, and properly set, an actual evaporation of 10.5 to 1 is not unusual.

During the year 1885, experiments were conducted, in France, for the purpose of ascertaining the best method of burning the less volatile portions of Russian petroleum, and of determining its evaporative efficiency. The hydro-carbon, in liquid form, and known in France as "Astatkis," is thrown into a properly prepared furnace in a spray, where it is perfectly burned, without smoke. The results have been in the highest degree satisfactory.

In one set of experiments, the rate of combustion ranged from 5.2 lbs. to 8.6 lbs. per square foot of grate per hour; while the evaporation, per pound of fuel, ranged from 12.75 lbs. to 13.42 pounds. The boiler pressure was 6.6 lbs., and the temperature of the feed ranged from 71.6° to 75.3° Fahr.

The evaporation, reduced to 212°, ranged from 14.83 lbs. to 15.78 lbs. of water per pound of combustible.

In a second series of experiments, the rate of combustion ranged from 11.76 lbs. to 14.7 lbs., per square foot of grate, per hour; while the evaporation, per pound of fuel, ranged from 11.57 lbs. to 12.82 lbs. The boiler pressure was 6.6 lbs. and the temperature of the feed ranged from 73.4° to 82.4° Fahr.

The evaporation, reduced to 212°, ranged from 12.15 to 14.85 lbs. per pound of fuel.

The ratio of the heating surface to the grate surface was 24.32 to 1.

With a larger heating surface, and with better proportions otherwise, it is likely that a still higher rate of evaporation would have been realized.

48. *Results and Deductions from Boiler Tests.*—

a. Boiler of W. & L. E. Gurley, Troy, N. Y., designed by the writer and tested March 27th, 1882.

PROPORTIONS OF BOILER.

Ratio of grate to calorimeter	8 to 1
" " " area over bridge-wall	9.7 to 1
" " heating surface to grate	44 to 1

RESULTS OF TEST.

Lbs. of water evaporated	20961.
" " coal burned	1940.
Temperature of feed-water	112.°
Boiler pressure, pounds	75.
Lbs. of water evaporated per pound of coal	10.8
Equivalent evaporation, from and at 212°, per pound of coal	12.295

ANALYSIS OF COAL.

Sulphur	0.892%
Moisture	1.955%
Carbon	88.188%
Ash	8.965%
	100.000%

Specific gravity, 1.46.

Calorific power, 13477 heat-units.

(*b.*) Battery of three cylindrical flue-boilers which furnished steam for duty-test of pumping engines at Allegheny City, Pa, Aug. 30th to Sept. 2d, 1884.

PROPORTIONS OF BOILERS.

Ratio of grate to calorimeter	10.296 to 1
" " " " cross area of chimney	7.47 " 1
" " heating surface to grate	19.796 " 1
Height of chimney, above grate	59. feet.

RESULTS OF TEST.

Lbs. of water evaporated, 24 hours	115000.
" " bituminous coal burned	16575.
Mean temperature of feed-water	107.53°

Mean steam pressure..........................	108.38
Mean rate of coal consumption, in pounds, per square foot of grate, per hour..............	10.464
Lbs. of water evaporated, per lb. of coal........	7.632
" " steam per net horse power, per hour.....	18.568

(*c*). Battery of two cylindrical flue boilers of Decatur, Ills., water-works, 1884.

PROPORTIONS OF BOILERS.

Ratio of grate to calorimeter................ ..	3.986 to 1
" " heating to grate surface..............	25.191 to 1
" " grate to cross area of chimney flue....	2.391 to 1
Diameter of chimney, feet.....................	4.5
Height " " "	91.5

RESULTS OF TEST.

Lbs. of coal burned, in 16 hours....................	6000.
" " water evaporated, "	27642.
" " " " per lb. coal...............	4.607
Equivalent evaporation, from and at 212°............	5.534
Lbs. coal burned, per square foot of grate, per hour..	9.86
" of steam, per horse-power, per hour............	25.35
" " coal, per net horse-power, per hour..........	5.21
Per cent. of water entrained in the steam............	5.33

(*d*.) Battery of two cylindrical tubular boilers of Saratoga, N. Y., water-works, 1882.

PROPORTIONS OF BOILERS.

Ratio of grate to calorimeter..................	8.08 to 1
" " heating surface to grate..............	44.655 to 1
" " grate to cross-area of chimney........	5.51 to 1
Height of chimney, feet......................	83.5

RESULTS OF TEST.

Lbs. of water evaporated, in 20 hours...............	72678.
" " coal burned, " " "	6750.
Temperature of feed-water.........................	175°.
Boiler pressure, pounds............................	75.
Lbs. of water evaporated, per lb. of coal.............	10.768
" " coal burned, per square foot of grate, per hour	5.92
" " steam per net horse-power, per hour.........	19.015
" " coal, per net horse-power, per hour..........	1.766
Per cent. of water entrained in the steam............	4.43

The Saratoga boilers are of the same type, proportions and general arrangement, as that of the Messrs. Gurley; and, it will be observed, their evaporative efficiency is almost exactly the same.

49. *Work of a Pound of Steam.*—In determining the work due to a pound of steam—used without expansion—we shall make use of the old and familiar diagram, Fig. 11, which represents an ideal apparatus, in which *a b f e* is a cylinder, of indefinite length, having a transverse sectional area of one square foot, closed at the bottom and open at the top In this cylinder is placed a piston, which is supposed to move without friction, and which is exactly balanced, in all positions, by the weight *w*, attached to a cord, connected with the piston, and passing upward and over the pulleys as indicated. Let a cubic foot of water, at the temperature of maximum density (39.1°), be placed in the bottom of the cylinder, and let heat be applied until the water is wholly converted into steam.

The piston will have been raised through a vertical height, V feet, against the atmospheric pressure, p pounds per square inch, and a quantity of work, W, will have been performed which will be expressed thus :

$$W = 144\, p\, V^{*} \text{ foot-pounds} \ldots\ldots\ldots\ldots\ldots (a)$$

If this work has been performed in *one hour*. the power, W', in horses, will be expressed as follows :

$$W' = \frac{144\, p\, V}{60 \times 33000} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

But one cubic foot of water, at 39.1°, weighs 62.425 pounds. Dividing, then, (a) and (b) by 62.425, we obtain the work and power due to *one pound of water converted into atmospheric steam, in one hour.*

Thus (a) becomes

$$W_1 = \frac{144\, p\, V}{62.425}$$

$$= 2.306\, p\, V \text{ foot-pounds;}$$

*We do not overlook the fact that, strictly, $V - 1 +$ should be used. V is used because it is believed that in all cases, in practice, the uncertainty in the value of V is greater than 1.

and (*b*) becomes

$$W_1' = \frac{144 \times 14.7 \times 1641.4}{60 \times 62.425 \times 33000}$$
$$= 0.028 \text{ horse-power.}$$

It follows, then, that in order to maintain one horse-power, for an hour, under the conditions stated, there will be required

$$\frac{1}{W_1'} = \frac{1}{0.028} = 35.7 \text{ pounds of steam.}$$

50. *Efficiency of the Heat Expended.*—Under the conditions assumed in Art. 49, the total heat of evaporation, per pound of water, is practically,

$$966 + (212 - 39) = 1139$$

units; the mechanical equivalent of which is

$$1139 \times 772 = 879{,}338 \text{ foot-pounds.}$$

This is equivalent to

$$\frac{879{,}338}{60 \times 33000} = 0.4441$$

horse-power, if the heat is expended in an hour.

The work actually performed by the steam is 0.028 of a horse-power; the evaporation being effected in one hour.

Now, designating the ratio of the work actually performed, to the work due to the expenditure of heat, the efficiency of the heat—or, which is perhaps better, the *efficiency of the apparatus*—by which the work is performed, we have this

$$\text{Efficiency} = \frac{0.028}{0.4441} = 0.063.$$

Thus it appears that our apparatus, Fig. 11, is a most inefficient and wasteful machine. We shall learn, later, Art. 57, that even the best modern steam engine is not very much better.

51. *Pounds of Coal per Horse-Power, per Hour.*—It has been shown, Art. 28, that the number of heat-units which may be utilized in the boiler, from the combustion of a pound of anthracite coal—neglecting the loss due to radiation—varies from 11288 to 12647.

It may be assumed that, under the most favorable conditions, 11400 heat-units may be utilized from the complete combustion of a pound of anthracite coal.

The total heat of evaporation, from the temperature of maximum density and at 212° is, in round numbers, 1140 units.

Then, since under the conditions of Art. 49, 35.7 lbs of steam are required in order to maintain one horse-power for one hour, it follows that the required weight of coal, per horse-power, will be

$$C_h = \frac{35.7 \times 1140}{11400} = 3.57 \text{ pounds.}$$

It is to be understood that the values 35.7 and 3.57 represent the water and coal under ideal conditions only, and that they represent, under such conditions, the water and coal necessary to maintain an absolute, or total, horse-power for one hour.

Their principal use is for purposes of comparison with the results of actual practice, and to enable us to form a just estimate of the extent of the improvements, in the efficiency of the steam engine, which have been effected by modern methods.

The following table contains some of the results of a trial of the machinery and boilers of the U. S. steamer "Dispatch," made in 1881:

Designation of Power.	No. of Horse-power.	Pounds of Coal, per Horse-power, per hour.
Total power,	493.74	3.1392
Indicated power,	402.80	3.8481
Net power,	370.40	4.1844

In this case, the cost of a total horse-power is only 12% less than the estimated cost, when steam is used in the apparatus of, and in the manner indicated in, Art. 49.

It may, perhaps, be fairly stated that, in the most efficient modern engines, the cost of the total power, per horse-power per hour, is not materially less than two pounds.

52. *The Total Useful Work of Steam, as Affected by Pressure and Condensation.*—In Fig. 11, let the sectional area of the cylinder be one square inch, and suppose the water to occupy one foot of its length, at the bottom. Let it be assumed, too, that when the water is converted into steam, under different pressures, the resulting volumes of steam will be inversely proportional to the pressures; in other words, let it be assumed that $p\,v$ is constant.

This assumption, while not strictly true, is sufficiently near the truth for our present purpose.

If, now, the water be converted into steam, under a pressure of one atmosphere, the work will be wholly performed in overcoming the resistance of the atmosphere, and no useful or effective work will result; but if a jet of water be introduced, the steam will be condensed, a vacuum will be formed, the piston will be forced down by the atmospheric pressure p, and an equivalent load placed upon w, will be raised through a height v. In raising this weight, useful work will be performed, which will be measured by $p\ v$, and which will be the total useful work performed under the assumed conditions.

Next, let the water be converted into steam of two atmospheres—a load equal to p being placed on top of the piston. The piston, in this case, will rise to a height of $\frac{1}{2}v$ feet. The total work performed will be

$$2p \times \tfrac{1}{2}v = p.v.$$

One-half of this work will be expended in overcoming the resistance of the atmosphere, and the other half, which will be expended in raising the added weight p through the height $\frac{1}{2}v$, and which will be measured by $\frac{1}{2}p.v.$, will be useful work.

Introducing a jet of water, condensing the steam, forming a vacuum under the piston as before, and at the same time transferring the added weight p, from the piston to the weight or counterpoise w, the piston will be forced down by the atmospheric pressure, and the weight p will be raised to a height $\frac{1}{2}v$. During this part of the operation, then, useful work will be performed, as the result of the condensation, which will be measured by

$$p \times \tfrac{1}{2}v = \tfrac{1}{2}p.v.$$

In this case, then, the total useful work will be

$$\tfrac{1}{2}p.v. + \tfrac{1}{2}p.v. = p.v.$$

Again, let the water be converted into steam of three atmospheres—first placing a weight or load of $2\ p$ on the top of the piston.

Now, during the conversion of water into steam, the piston, with its load of $2\ p$, which, together with the atmosphere will constitute a load of $3\ p$, will be raised through a height of $\frac{1}{3}v$ feet, and a total work

$$3p \times \tfrac{1}{3}v = p\ v$$

will be performed.

The useful work will, however, be

$$2p \times \tfrac{1}{3}v = \tfrac{2}{3}p.v.$$

Removing now the load, $2p$, from the piston, placing a load p on the counterpoise w, and again condensing the steam, the piston will be forced down by the atmosphere and the weight p, added to the counterpoise, will be raised through a height $\frac{1}{3}v$.

The useful work, therefore, resulting from the condensation, will be

$$p \times \tfrac{1}{3}v = \tfrac{1}{3}p\,v;$$

while the total useful work, resulting from the entire operation will be

$$\tfrac{2}{3}p\,v + \tfrac{1}{3}p\,v = p\,v,$$

as before.

In a similar manner we may determine the useful work due to the formation of steam under any pressure, together with its subsequent condensation.

The results are collected and placed in the following table, in which p represents a pressure of one atmosphere and v, the specific volume of steam of one atmosphere:

Pressure of Steam.	Useful Work Performed.		
	During Formation of the Steam.	During Condensation.	Total.
p.	0.	$p.\ v.$	$p.\ v.$
$2p$.	$\frac{1}{2}p.\ v.$	$\frac{1}{2}p.\ v.$	$p.\ v.$
$3p$.	$\frac{2}{3}p.\ v.$	$\frac{1}{3}p.\ v.$	$p.\ v.$
$4p$.	$\frac{3}{4}p.\ v.$	$\frac{1}{4}p.\ v.$	$p.\ v.$
....	.....	.....	....
$n.p$	$\frac{n-1}{n}p.\ v.$	$\frac{1}{n}p.\ v.$	$p.\ v.$

These results show that the total useful work of steam, under the assumed conditions, or the absolute work, under varying pressures, is constant, and is measured by $p.\ v.$

The same conclusions would have been reached had we assumed a vacuum in the cylinder above the piston. In that case the total work would have been determined at once, and would have been $p.\ v.$, $2p \times \frac{1}{2}v = p.\ v.$, $3p \times \frac{1}{3}v. = p\,v$, etc, etc., as expressed in the last column of the foregoing table.

The method which has been employed was adopted for a twofold purpose:

First—To accustom the student to study the action of steam in a cylinder, and

Second—To separate the effect of condensation, and to make it appear—as it does, clearly, in the table—that the gain in work

due to condensation diminishes, both absolutely and relatively, very rapidly, as the pressure of the steam is increased.

This fact being established, it follows, necessarily, that at some pressure the gain in power due to condensation must become so small that condensation is no longer profitable: in other words, that the value of the power thus gained is less than the cost at which it is secured.

In order to show how far this conclusion is borne out by practical results obtained from modern engines, the results of careful trials of a high pressure non-condensing engine, and a high pressure condensing engine, together with deductions from the same, are presented.

These are taken from a discussion of the subject of "Economy of Condensation," by Chief Engineer B. F. Isherwood, U. S. N., in the *Journal* of the Franklin Institute, in May, June and September, 1881, and consist of results from a non-condensing engine tested at Mulhouse, Alsace, and from a condensing engine, tested by J. W. Hill, at the Cincinnati Millers' Exposition, in June, 1880.

DATA.		RESULTS.	
		Condens'g.	Non-Cond.
Diameter of cylinder, inches		24	16.06
Length of stroke, "		48	48
Clearance, per cent		2.46	2.91
Pressure steam in boiler, above atmos., lbs.		66.59	70.477
Cut-off at—decimal of stroke		0.10	0.2066
Steam expanded, times		9.9	4.37
Pressure in condenser, lbs		2.04	......
Temperature of feed-water, Fahr		100°	200°
Mean back-pressure, lbs		3.11	15.9
Pressures to work engine, *per se*, lbs		2.164	1.89
DEDUCTIONS.			
Horse-power	Total	143.71	116.67
	Indicated	127.30	74.32
	Net	115.89	68.80
Pounds of water or steam, per horse-power, per hour	Total	23.04	18.74
	Indicated	26.02	29.42
	Net	28.58	31.79
Units of heat expended, per horse-power, per hour	Total	25569.17	18922.34
	Indicated	28866.42	29703.55
	Net	31707.07	32091.61

The final values in the foregoing table indicate that in these two cases a net, effective, or commercially valuable horse-power was obtained at practically the same expenditure of heat.

Thus it appears, under the conditions of these two trials, that the limit of economy of condensation is reached at a steam-pressure of from 70 to 75 pounds per square inch above the atmosphere.

It will be interesting to note that, in the case of the non-condensing engine the difference between the water evaporated in the boiler, per hour, and that accounted for in the cylinder, at the point of cut-off, was 614.7966 lbs. or 28.1 per cent. of the former.

At the end of the stroke, the water accounted for was 219.58 pounds, or 10.04 per cent. less than that evaporated in the boiler.

These results mean that over 28 per cent. of the steam was condensed before the piston reached the point of cut-off, and that more than one-third of the water of condensation was re-evaporated during expansion, or while the piston was completing its stroke.

Notwithstanding, however, these large losses due to condensation and re-evaporation, the net horse power of this engine was obtained at almost exactly the same original expenditure of heat as was that of one of the most efficient of modern simple condensing engines.

IV. The Steam Engine—Early Types.

53. The earliest type of steam engine was that designed by Captain Savery, near the close of the seventeenth century, which is known as Savery's engine, and its general plan and operation are indicated in Fig. 12.

A is a hollow vessel, to the bottom of which is attached a pipe *P*, leading to the water in the well *W*. Steam is admitted to the vessel *A*, through the pipe and cock c; air escapes from the vessel *A* through the cock c_3; water enters *A*, through the pipe *P* and cock c_1, and is discharged into the tank w, through the cock c_2.

The operation of the engine is as follows: The cocks c and c_3 being open and c_1 and c_2 being closed, steam enters the vessel *A*, through c, and air escapes through c_3, until the vessel *A* is filled with steam. c and c_3 are then closed and c_1 is opened. As the steam in *A* condenses, the water rises from *W* through the pipe

P, and cock c_1, and fills *A*. c_1 is then closed, and c_3 and c_2 opened and the water in *A* is discharged into the reservoir *w*.

The vessel *A* is now filled with air and its walls are cold.

c_2 is then closed, and *c* and c_3 opened: steam enters *A*, expelling the air through c_3 until *A* is filled with steam; *c* and c_2 are again closed and *c* opened; when the vessel *A* is again filled with water, which is discharged into *w* as before. Thus the operation is continued.

Water can thus be raised about 33 feet—the height at which the column becomes equivalent to the atmospheric pressure.

Much loss of heat results from the condensation of the steam in contact with the cold walls of *A*, as it is successively filled and discharged.

54. The second type, designed by Newcomen, early in the eighteenth century, is known as Newcomen's engine, and is represented in Fig. 13.

B F is an iron cylinder, open at the top, in which works a piston *E*, which is loosely packed with hemp, to prevent the escape of steam from below. A small quantity of water was kept above the piston to aid in preventing leakage of air or steam. To the end of the piston-rod is attached a rope or chain, leading to an arc on the end *K*, of the beam *K L*, which vibrates about a centre *O*, and which is provided with a similar arc at the pump end *L*. The pump-rod, and its counter-weight *I*, are suspended by a rope or chain from this second arc, at the point *H*.

A is the steam valve, *c* the injection cock, or valve, and *s* is a snifting valve, attached to the extremity of an exhaust pipe, leading from the bottom of the steam cylinder to the hot-well *D*. *J* is a tank from which injection or condensing water is supplied, through *c*, to the steam space below the piston *E*.

Steam having an absolute pressure of one atmosphere was used.

The weight of the pump-rod and its counterpoise *I*, was so adjusted that the excess of their weights over that of the piston and its rod, was a little less than the pressure of the atmosphere on the top of the piston *E*. The pump-rod extended downward to a lifting and forcing pump at the bottom of the well, which was, in some cases, at a very great depth below the surface.

To put the engine in operation, steam was admitted, through the cock *A*, to the cylinder, until the air was forced out through the snifting-valve *s*. The final expulsion of the air from the

space *B* below the piston, was indicated by the escape of *steam* from the snifting-valve.

Under this condition of things, the pressures below and above the piston were exactly equal, and the unbalanced excess of weight of the pump-rod and its counterpoise *I*, over that of the piston and its rod, was expended upon the pump below, in forcing its charge of water to the surface.

The steam piston, therefore, rose to the top of its stroke, and the cylinder was filled with steam. The injection cock *c*, was then opened, and a sufficient quantity of water, from the tank *J*, was admitted to the cylinder to condense the steam, thus forming a vacuum in the cylinder below the piston. Now the pressure of the atmosphere on the top of the piston, is a little greater than the unbalanced weight of the pump-rod and its counterpoise *I*. This excess of atmospheric pressure causes the piston to descend, and the pump-rod to rise and charge the pump with water.

The excess of the atmospheric pressure over the unbalanced weight of the pump-rod and its counterpoise, was made just equal to the excess of the latter over the weight of the column of water which was to be raised during each descent of the pump-rod. Thus, the unbalanced forces, during the descent of the pump-rod, and during the descent of the steam piston, being equal, the motion of the engine was nearly uniform.

At the outset the valves were operated by hand, and the motion was necessarily slow. In 1713 a boy, named Humphrey Potter, who was employed to manipulate the valves, devised a combination of rods and pins which rendered the operation of the valves automatic, and materially increased the speed of the engine.

The loss of heat was, however, still great; owing to the successive heating and cooling of the walls of the steam cylinder.

55. With a view to remedying this obvious defect, James Watt, about the year 1778, devised his single acting engine, with a separate condenser; which, like its predecessors, was used for pumping water from mines.

The general arrangement of Watt's engine is shown in Fig. 14.

P is the piston, moving in a steam cylinder, which is closed both at the top and at the bottom; *K* is the steam-pipe, which turns downward, along the cylinder, and extends as far as the lower end of the cylinder; *W* is the condenser, *f* the foot-valve,

H the air-pump, and *I* the hot-well. From the hot-well, a portion of the water goes to the boilers, and the remainder is wasted.

D and *E* are steam-valves, and *F* is the exhaust valve. Atmospheric or low-pressure steam was used in this engine, and similar relations existed between the weights of the piston, pump-rod and the load on the pump, to those indicated in the case of the Newcomen engine.

To operate the Watt engine, open the valves *D*, *E* and *F*, and blow through, or expel the air from the cylinder, into the condenser—through a snifting valve or its equivalent.

During this operation the pressures above and below the piston, will be the same; and the piston will rise, on account of the slight preponderance of the pump-rod—at the other end of the beam—over its load, to the top of the cylinder.

As soon as steam escapes from the cylinder—indicating that the air has been wholly expelled—the valves *D* and *E* are closed, and the injection *i*, opened; the injection water, entering the condenser, condenses the steam which fills the cylinder, and forms a vacuum below the steam-piston. The exhaust valve *F*, is then closed, and the steam-valve *D* opened; steam enters above the piston—*E* being closed—and forces it down to the bottom of the cylinder; *D* is then closed, and *E* opened. There being now free communication between the spaces above and below the piston, the steam will fill both spaces, and the upward and downward pressures on the piston will be equal. The piston will then rise—owing to the preponderance of weight at the other end of the beam—and the steam will pass from the space above the piston to that below it. As soon as the upward stroke is completed, the valve *E* is closed, and the injection valve *i* opened; the steam then passes to the condenser and is condensed. A vacuum is thus formed below the piston. The exhaust *F* is then closed, and the steam-valve *D* opened. By repeating the operation of the valves, the motion of the engine is made continuous. The air-pump rod *R*, is connected to the vibrating beam overhead; so that the air-pump bucket is raised, and falls, with the steam-piston. The water in the condenser passes through the foot-valve *f*, and stands at the same elevation in the condenser and in the air-pump cylinder. When, therefore, the air-pump piston descends and reaches the level of the water in its cylinder, the foot-valve is closed, the valves in

the air-pump piston open, upward, and a charge of water passes through, into the space above the piston. As the piston rises, its valves close, and the charge of water is carried to the top of the air-pump, where a floating cover is raised, and the charge of water flows over into the hot-well *I*. The air-pump is, in this case, single-acting, and a charge of water is drawn from the condenser with each upward stroke of the steam-piston.

The engine just described is also single acting, as were the engines of Savery and Newcomen.

56. In 1780, Watt constructed a double-acting engine, with a crank and fly-wheel. This differed from his single-acting engine only in the addition of a second cylinder, which was placed directly under the first; a single piston-rod serving for both, and being so connected with the steam end of the working beam, that the steam acting on the top of the piston of the first, or upper cylinder, forced the end of the working beam downward, while the steam acting on the bottom of the piston of the second cylinder, forced the end of the beam upward.

The engine, thus modified, possessed all of the essential elements of the modern steam engine. Its power, and the necessary expenditure of steam, together with the necessary capacities of the condenser and air pump were, of course, twice as great as in the case of the single acting engine.

In 1782, Watt constructed a 40 horse-power, non-expansive engine, from which a horse-power was maintained by the combustion of $8\frac{1}{4}$ pounds of coal per hour, and by the evaporation of 0.674 of a cubic foot, or about 43 pounds, of water per hour.

When the steam was expanded 1.518 times, the cost, per horse-power per hour, was reduced to 6.25 pounds of coal, and 0.501 of a cubic foot, or about 31 pounds, of water, respectively.

V. Jet and Surface Condensation.

57. *Quantity of Water Required for Condensation.*—Having reached the stage of development of the steam engine at which condensation was employed, in one of its present forms, it is proper to explain the method of ascertaining the quantity of condensing water required in any case.

For this purpose let

w = the number of pounds of steam to be condensed per minute.

T = the total heat of the steam, per pound, in heat-units.
t = the temperature of the injection or condensing water.
t' = the temperature of the water of condensation, or of the hot-well.
W = the number of pounds of condensing water required, per minute.

Now, the number of heat-units to be taken from the steam, per minute, will be

$$w(T - t');$$

while the heat which will be taken up by the condensing water, in the same time, will be

$$W(t' - t).$$

Since these two quantities of heat must be equal, we have

$$W(t' - t) = w(T - t');$$

whence,

$$W = w\frac{T - t'}{t' - t}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Example.—If $w = 40$ pounds, and the pressure be 75 pounds above the atmosphere, and if t and t' be 40° and 100°, respectively, we shall have, Art. 38,

$$T = 1179.6 + 32 = 1211.6 \text{ units.}$$

Then, (29),

$$W = 40 \times \frac{1211.6 - 100}{100 - 40}$$

$$= 741.08 \text{ pounds.}$$

If $w = 1$ pound,

$$W = 18.527 \text{ pounds.}$$

If, all the other conditions being the same, and the temperature of the injection water (t) be 70°, we have

$$W = 40 \times \frac{1211.6 - 100}{100 - 70}$$

$$= 1482.16 \text{ pounds.}$$

If $w = 1$ pound,

$$W = 37.054 \text{ pounds.}$$

58. *Jet and Surface Condensers.*—Where the injection or condensing water is admitted directly to, and mingles with, the

steam to be condensed, the condenser is termed the *jet* condenser. In this case, however, since the weight of the warm water, passing to the air-pump and hot-well, is from 18 to 37 times as great as that which results from the condensation of the steam, it follows that from $\frac{17}{18}$ to $\frac{36}{37}$ of the heat imparted to the condensing water is wasted and wholly lost.

In the first part of the example of Art. 57, $100 - 40 = 60$ units of heat are communicated to each pound of the injection water; of this, $\frac{1}{18}$, or $3\frac{1}{3}$ units, are returned to the boiler, in the feed-water, and $\frac{17}{18}$, or $56\frac{2}{3}$ units, are lost in the water of condensation, which is wasted from the hot-well.

Each pound of steam contains $1211.6 - 100 = 1111.6$ heat-units more than it contained when it entered the boiler as feed-water; of this, only 60 units, or about 5.4 per cent. is saved, and returned to the boiler; while $1111.6 - 60 = 1051.6$ units, or 94.6 per cent. are wasted from the hot-well.

As a matter of fact, only a part of the heat of the steam which leaves the boiler, remains in it when it reaches the condenser. This part, however, is a very large part; the losses being the comparatively small ones, due to radiation and to the work done by the steam.

The expenditure of heat due to the work done, probably ranges from 6.5 to 12.5 per cent., and depends upon the manner in which the steam is used. (See articles 7, 45, 50 and 68).

When the feed-water is salt, or sea-water, the loss is greater. The average saturation of sea-water being about $\frac{1}{32}$—one pound of salt in 32 pounds of sea-water—it follows that, if the evaporation were continued long enough, the boiler would become filled with salt. To avoid this, with the jet condenser, the evaporation is only continued to the point at which the saturation becomes doubled, or $\frac{2}{32}$. This occurs when one-half of the water has been evaporated, leaving the salt in the other half.

The doubly saturated water is then blown out of the boiler and a new supply added.

Practically, the operation is managed as follows:

If Q be the quantity of water in the boiler at the beginning, additional water is pumped in, to supply the loss by evaporation, until the quantity added is equal to Q. The salt due to a quantity $2\,Q$ is now contained in a quantity Q, whose saturation is $\frac{2}{32}$, or double the normal saturation.

The blow-off valve is now so adjusted as to permit water to

be blown from the boiler at a rate equal to the rate of evaporation, and the rate at which the water is supplied to the boiler—which has thus far been just equal to the rate of evaporation—is doubled.

In other words, one-half of the water which is supplied to the boiler is converted into steam, and the other half is blown out, at the temperature of the boiler; carrying with it a large quantity of heat, which is, of course, wasted or lost.

The loss of heat by blowing-off, in any case, may be readily determined, as follows:

Let n = the saturation of the water in the boiler, in 32nds.
H = the total heat of the steam above zero.
t = the temperature of the feed-water.
t_1 = the temperature of the water and steam in the boiler.
L = the loss, per cent., of heat by blowing off.

Then, of the quantity of water supplied to the boiler, $\frac{1}{n}$th will be blown off, at a temperature t_1, and $1 - \frac{1}{n}$th will be converted into steam.

The heat expended in converting $1 - \frac{1}{n}$th of the feed into steam, will be, for each pound,

$$\frac{n-1}{n}(H-t);$$

while the heat expended in raising the temperature of $\frac{1}{n}$th of the water, from the temperature t, of the feed, to the temperature t_1, of the boiler, will be

$$\frac{1}{n}(t_1-t);$$

which represents the loss, in heat-units, due to each pound of feed-water supplied to the boiler.

The total expenditure of heat, per pound of feed-water, will therefore be

$$\frac{n-1}{n}(H-t)+\frac{1}{n}(t_1-t);$$

and the loss, per cent., of heat

$$L=\frac{\frac{100}{n}(t_1-t)}{\frac{n-1}{n}(H-t)+\frac{1}{n}(t_1-t)}$$

$$=\frac{100\,(t_1-t)}{(n-1)\,(H-t)+(t_1-t)}. \qquad \text{.............. (30)}$$

Example.—Required the loss per cent. of heat, when the saturation is carried at $\frac{3}{32}$; the temperature of the feed being 100° and the pressure of the steam being 25 pounds above the atmosphere. Here, $n = 3$; $t = 100$; $t_1 = 267.17$ and $H = 1163.43 + 32 =$ 1195.43 units.

Then, (30),

$$L = \frac{100\ (267.17 - 100)}{2\ (1195.43 - 100) + (267.17 - 100)} = 7.09.$$

If the pipe through which the salt water is blown off be formed into a coil, and passed through the mass of feed-water, on its way from the hot-well to the boiler, the temperature of the latter will be raised through something less than one-third of the difference between 100° and 267.17°. Let us suppose it to be raised from 100° to 150°, Then, in the foregoing example, the loss, per cent., of heat will be reduced from 7.09 to

$$\frac{100\ [267.17 - 100 - 1\ (150 - 100)]}{2\ (1195.43 - 100) + 267.17 - 100 - 3\ (150 - 100)} = 5.3$$

The recovery of heat by the feed-water heater in this case, therefore, appears to be

$$\frac{100\ (7.09 - 5.3)}{7.09} = 25.2 \text{ per cent.}$$

of that which otherwise would have been lost.

The ***surface condenser***, Fig. 15, is designed, not only to prevent a large portion of the loss which results from blowing off, but to prevent deposits, mainly of the salts of lime, upon the inner surfaces of the boiler. It consists of a large number of composition tubes, about $\frac{5}{8}$ of an inch in diameter, through which cold water is kept constantly moving, by circulating pumps, and around which the steam is exhausted and condensed by the cool surfaces.

Were there no leakage, the water resulting from the condensation of the steam, could be returned to, and wholly supply, the boiler; in which case the boiler, once supplied with fresh water, could be kept in operation indefinitely, without an additional supply.

On account of unavoidable leakage, however, small quantities of salt water must occasionally be added, in order to maintain the supply in the boiler.

Messrs. Long and Buel, in speaking of the relative merits of the two classes of condensers, say: "In practice the gain by the use of surface condensers is not as large as the theoretical result. Nearly all condensers leak to some extent, so that salt water mingles with the water of condensation. Moreover, all the water that is evaporated by the boilers is not preserved in the condenser, so that a salt feed is necessary from time to time. Surface condensers are also much heavier, and occupy about twice as much room as the old jet condensers. The first cost, and the numerous repairs they require, must be considered. The vacuum also is not generally as good as that produced with jet condensers. It has been found, too, that surface condensers have a powerful influence in causing the corrosion of boilers. Still, with all these drawbacks, the advantages of surface condensers have been considered so great that they have been largely introduced."

59. *Condensing Surface Required.*—In connection with the surface condenser, arises a question as to the rate of transmission of heat through the material of the condensing tubes, and as to the extent of condensing surface required.

Upon this subject we are without any very precise information, and engineers are compelled to rely, very largely, upon the results of experience.

In the American Ordnance Manual, we find the following:

$$q = \frac{T - T'}{g\,x}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

in which $T =$ the temperature on one side of the material.

$T' =$ the temperature on the other side.

$x =$ the thickness of the metal in inches.

$g =$ an experimental constant.

$q =$ the number of heat-units transmitted per hour, per square foot of surface.

Example.—If 25 pounds of steam be used, per horse-power per hour, and 1100 heat-units are to be transmitted per pound of steam, the total number of heat-units to be transmitted, per horse-power per hour, will be $25 \times 1100 = 27500$.

If, again, the material be of composition, 0.06 of an inch thick, and g for this material be 0.0042, we shall have for the requisite area of condensing surface, in square feet,

$$A = \frac{27500}{q} = \frac{27500\,g\,x}{T - T'}.$$

Putting $T = 212°$ and $T' = 80°$ this becomes

$$A = \frac{27500 \times 0.0042 \times 0.06}{212 - 80} = 0.052.$$

If the condensation be effected in one-twentieth of the time, so that the vacuum will be wholly effective during the other nineteen-twentieths, this surface must be twenty times as great. This will give $20 \times 0.052 = 1.04$ square feet per horse-power. If now the heating surface be equal to 35 times the grate surface, and if each square foot of grate surface represent 3.5 horse-power, the heating surface per horse-power will be $\frac{35}{3.5} = 10$ square feet.

Under these conditions, if the formula be correct, the condensing surface must be made equal to about *one-tenth* of the heating surface.

In the machinery of a steam vessel built for the navy during the late war, the areas of grate and heating surfaces were:

Grate surface, square feet............	546
Heating " " "	21840

In the surface condensers there were 2800—$\frac{5}{8}$ inch composition tubes, 10 feet long, having a thickness equal to No. 17 wire gauge. The area of the condensing surface was thus about 4575 square feet, and the ratio of the condensing surface to the heating surface,

$$\frac{4575}{21840} = \frac{1}{5} \text{ nearly.}$$

There were superheating boilers, which had heating surfaces, aggregating about 7 per cent. of the total heating surface of the main boilers.

In this case the condensing surface appears large; that, however, was objectionable only as it increased the cost, and required greater space.

One-tenth is the ratio adopted by some engineers.

VI. Work Performed in the Cylinder.

60. *The Work Done in the Cylinder.*—Fig. 16. The total and indicated* works done by the steam on one square inch of the piston in a steam cylinder may be represented, graphically, as follows:

*So called because it is measured by the indicator.

Let $C D' =$ the stroke of the piston $= s$.
$C A = p =$ the steam pressure above zero, per square inch.
$C F =$ one atmosphere $= 14.7$ pounds.
$A F = p - 14.7$ pounds $=$ gauge pressure.
$F E =$ atmospheric line

Then, if the steam pressure be maintained uniformly throughout the stroke, the total work of the steam, in foot-pounds, on each square inch of piston, will be measured by

$$C A \times C D = p. s.,$$

and will be represented by the area of the rectangle $A\ B\ D\ C$.

The work of the atmospheric resistance will be measured by

$$C F \times C D = 14.7\ s,$$

and will be represented by the area of the rectangle $C\ D\ E\ F$.

The *indicated* work of the steam, per square inch of piston, will be measured by

$$(A\ C - F\ C) \times C\ D$$
$$= A\ F \times C\ D = (p - 14.7).\ s, \quad \ldots\ldots\ldots\ldots(32)$$

and is represented by the area of the rectangle $A\ B\ E\ F$.

If a condenser be used, a portion only of the atmospheric resistance will be removed, and there will remain a small resistance, represented by $C\ c$, due to the tension of the uncondensed vapor in the condenser, which is measured in inches of mercury, as is the atmospheric resistance, which is measured by the barometric column.

Ordinarily, two inches of mercury are treated as representing a pound.

If, then, $b =$ the barometric column, in inches, and
$v =$ the vacuum, also in inches,

$\frac{1}{2}b$ will be represented by $F\ C$, and $\frac{1}{2}v$ by $F\ c$. The atmospheric resistance, removed by the partial condensation of the steam, will be represented by $F\ c = \frac{1}{2}v$ pounds, and the gain in net work by the area of the rectangle $E\ F\ c\ d$.

The measure of the gain, in net work, will be

$$\tfrac{1}{2}v.\ s.$$

In this case the indicated work will be equal to the sum of the indicated work when the steam is not condensed, and the work gained by condensation; or,

$$\text{Indicated work} = \left(p - \frac{b}{2} + \frac{v}{2}\right)s.$$

$$= \left(p - \frac{b-v}{2}\right)s \text{ foot-pounds.} \quad \ldots\ldots\ldots(33)$$

The work of the resistance due to the tension of the uncondensed vapor in the condenser, is represented by the area of the rectangle $C\,D\,d\,c$, and is measured by

$$\frac{b-v}{2}.\,s.$$

This work of the resistance in the cylinder, added to the indicated work, gives the *total* work

$$= \left[p - \frac{b-v}{2}\right].\,s + \frac{b-v}{2}.\,s$$

$$= p.\,s.$$

as in the first case.

It is to be kept constantly in mind, that the works which have been considered, are works performed in the cylinder, and that they are not, for the present, to be regarded as having any connection with external work.

61. *Gain in Work due, theoretically, to Expansion.*—Fig. 17. In order to illustrate the gain in work theoretically due to the expansion of steam in the cylinder, suppose $C\,D$ to be the stroke of the piston, and $D\,A$ to be the steam pressure, above zero. Suppose, also, that when the volume $A\,D\,F\,E$ of steam has been admitted to the cylinder, and the piston has reached the position $E\,F$, the steam valve be closed, so that no more steam can enter.

The steam will now expand, as the piston continues its stroke, with a constantly diminishing pressure, until the piston has completed its stroke, and until the steam has expanded into the volume $A\,B\,C\,D$, and entirely fills the cylinder. The final pressure of the steam is represented by $C\,K$.

The total work of the steam before expansion began, is represented by the rectangle $A\,D\,F\,E$, and is measured by $A\,D \times D\,F$.

The total work of the steam during expansion, or the gain in work due to expansion, is represented by the area $E\,G\,K\,C\,F$, and it is the measure of this work, performed without additional expenditure of steam, which we now seek.

In Fig. 17, let

$p = AD =$ initial steam pressure above zero, per square inch of piston.

$p' = GH =$ pressure, per square inch of piston when it reaches H.

$a = DF =$ the distance through which steam follows; or the distance traversed by the piston before the admission of steam to the cylinder is suppressed.

$l = DC =$ length of stroke of the piston.

$x = DH =$ portion of stroke completed when the pressure becomes p'.

The volumes of the steam in the cylinder when the piston is at F and H, being proportional to DF and DH, respectively, we have, by the law of Mariotte,

$$DH : DF :: p : p';$$

or

$$x : a :: p : p';$$

whence

$$ap = xp'; \qquad \text{(a)}$$

which is of the form $A^2 = xy$, and indicates that the curve EGK is an equilateral hyperbola, whose asymptotes are DA and DC.

From (*a*) we get

$$p' = \frac{ap}{x}.$$

If, now, the piston be supposed to move through the distance dx, beyond H, the corresponding work will be

$$p'dx = \frac{apdx}{x}$$

$$= p.a.d\log_e x;$$

which, being integrated between the limits $x = a$ and $x = l$, gives, for the total work done by the steam, during expansion,

$$\int_a^l p.a.d.\log_e x$$

$$= p.a.\log_e l - p.a.\log_e a$$

$$= p.a.(\log_e l - \log_e a)$$

$$= p.a.\log_e \frac{l}{a} \qquad (34)$$

The total work done before the beginning of the expansion being $p. a.$, it follows that the total work done during the entire stroke of the piston, is

$$p. a. + p. a. log_e \frac{l}{a}$$

$$= p. a. \left(1 + log_e \frac{l}{a}\right) \ldots\ldots\ldots\ldots\ldots (35)$$

It will be convenient, generally, to consider a as unity. If W represent the total work of the steam, per square inch of piston, per stroke, (35) may be written

$$W = p\left(1 + log_e \frac{l}{a}\right) \ldots\ldots\ldots\ldots\ldots (35^a)$$

$$= p_m l;$$

in which p_m = the mean pressure during the stroke.

We then have

$$p_m = \frac{W}{l}$$

$$= p. \frac{1 + log_e \frac{l}{a}}{l}. \ldots\ldots\ldots\ldots\ldots (36)$$

Log$_e$ indicates the Naperian or hyperbolic logarithm. It is found as follows:

$$log_e a. = \frac{\text{common } log. a}{\text{modulus com. system}} = \frac{log. a}{0.43429}$$

$$= 2.30258 \; log. a.$$

Example.—Required the hyp. log. of 3. The common logarithm of 3 being 0.477121, we have

$$log_e 3 = 2.30258 \times 0.477121$$

$$= 1.097 +$$

Ordinarily, it will be sufficient to multiply the common logarithm of the number by 2.3 simply.

The following table of logarithms will be found useful in estimating the theoretical work of the steam, in the cylinder, when the steam is used expansively:

TABLE OF HYPERBOLIC LOGARITHMS.

No.	Log.	No.	Log.	No.	Log.	No.	Log.
1.25	0.223	4.25	1.447	7.25	1.981	15.	2.708
1.5	0.405	4.5	1.505	7.5	2.015	16.	2.773
1.75	0.560	4.75	1.558	7.75	2.048	17.	2.833
2.	0.693	5.	1.609	8.	2.079	18.	2.890
2.25	0.811	5.25	1.658	8.5	2.140	19.	
2.5	0.916	5.5	1.705	9.	2.197		
2.75	1.012	5.75	1.749	9.5	2.251		
3.	1.100	6.	1.792	10.	2.303		
3.25	1.179	6.25	1.833	11.	2.398		
3.5	1.253	6.5	1.872	12.	2.485		
3.75	1.322	6.75	1.909	13.	2.565		
4.	1.387	7.	1.946	14.	2.639		

62. ***Mean Pressures.***—The following examples are given, in order to illustrate the mode of calculation, and to exhibit the results due, theoretically, to different measures of expansion.

First—Suppose that the steam follows 0.1 of the stroke. Then $a = 1$ and $l = 10$, and we have, (36),

$$p_m = p \cdot \frac{1 + \log_e 10}{10}$$

$$= p \cdot \frac{1 + 2.303}{10}$$

$$= 0.3303\, p.$$

Second—Let the steam follow 0.2 of the stroke. Then $a = 2$ and $l = 10$; or $a = 1$ and $l = 5$; in which, a being regarded as unity, l is expressed in terms of a; and (36),

$$p_m = p \cdot \frac{1 + \log_e 5}{5}$$

$$= p \cdot \frac{1 + 1.609}{5}$$

$$= 0.5219\, p,$$

Third—Let the steam follow 0.3 of the stroke.
Then, $a = 3$ and $l = 10$; or, making $a = 1$, $l = \frac{10}{3} = 3.33$, and

$$p_m = p \cdot \frac{1 + \log_e 3.33}{3.33}$$

$$= p \cdot \frac{1 + 1.204}{3.33}$$

$$= 0.6619\, p.$$

Fourth—Suppose that the steam follows 0.4 of the stroke. Then $a = 4$ and $l = 10$; or, a being made unity, $l = \frac{10}{4} = 2.5$, and

$$p_m = p \cdot \frac{1 + \log_e 2.5}{2.5}$$

$$= p \frac{1 + 0.916}{2.5}$$

$$= 0.7665\,p.$$

and similarly for other points of cut-off.

The values of p_m, for cut-off at each tenth of the stroke, have been calculated and are presented in the following table:

TABLE OF MEAN TOTAL PRESSURES, IN POUNDS PER SQUARE INCH.

Cut-off at	p_m	Cut-off at	p_m
0.1	0.3303 p	0.6	0.9065 p
0.2	0.5219 p	0.7	0.9494 p
0.3	0.6619 p	0.8	0.9785 p
0.4	0.7665 p	0.9	0.9948 p
0.5	0.8466 p	1.0	1.0000 p

Example.—Let the steam pressure be 75 pounds by the gauge, and let the cut-off be at 0.3 of the stroke.

Then
$$p = 75 + 14.7 = 89.7 \text{ pounds,}$$
$$p_m = 0.6619 \times 89.7$$
$$= 59.37$$

pounds above zero; or,

$$59.37 - 14.7 = 44.67$$

pounds above the atmosphere.

It should be distinctly understood that the values of p_m, given by the formula (36), or resulting from the use of the constants in the table, are pressures above zero, or absolute pressures.

63. *Absolute Theoretical Gain in Work due to Expansion.*—In Art. 62, we have shown the mean absolute pressures, due to different measures of expansion, to different points of cut-off, or to different periods of admission. In each of the ten cases presented in the table, different volumes of steam were supposed to be used. Now, in order to determine the absolute gain in work, in each case, due to expansion, let the area of the piston, the stroke of the piston, and the volume of the cylinder, each be unity.

The work, then, in each case will be represented by

$$1 \times 1 \times p_m = p_m.$$

Then, when steam follows full stroke, $p_m = p$, and we have for the total work

$$W_1 = p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

When the steam follows 0.9 of the stroke, 0.9 of a unit of steam will be used per stroke, and the total work of a unit, compared with (*a*), will be

$$W_{0.9} = p_m \times (1 + \tfrac{1}{9}) = \tfrac{10}{9} p_m$$
$$= \frac{10 \times 0.9948}{9} . p$$
$$= 1.105\, p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

When the steam follows 0.8 of the stroke, 0.8 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.8} = p_m \times (1 + \tfrac{1}{4}) = \tfrac{5}{4} p_m$$
$$= \frac{5 \times 0.9785}{4} . p$$
$$= 1.223\, p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

When the steam follows 0.7 of the stroke, 0.7 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.7} = p_m \times (1 + \tfrac{3}{7}) = \tfrac{10}{7} p_m$$
$$= \frac{10 \times 0.9494}{7} . p$$
$$= 1.356\, p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

When the steam follows 0.6 of the stroke, 0.6 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.6} = p_m (1 + \tfrac{2}{3}) = \tfrac{10}{6} p_m$$
$$= \frac{10 \times 0.9065}{6} . p$$
$$= 1.511\, p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (e)$$

When the steam follows 0.5 of the stroke, 0.5 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.5} = 2p_m = 2 \times 0.8466\,p$$
$$= 1.693\,p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (f)$$

When the steam follows 0.4 of the stroke, 0.4 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.4} = p_m\left(1 + \tfrac{3}{2}\right) = 2.5\,p_m$$
$$= 2.5 \times 0.7665\,p$$
$$= 1.916\,p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (g)$$

When the steam follows 0.3 of the stroke, 0.3 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.3} = p_m\left(1 + 2\tfrac{1}{3}\right) = 3\tfrac{1}{3}\,p_m$$
$$= \frac{10 \times 0.6619}{3} \cdot p$$
$$= 2.206\,p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (h)$$

When the steam follows 0.2 of the stroke, 0.2 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.2} = p_m\left(1 + \tfrac{8}{2}\right) = 5\,p_m$$
$$= 5 \times 0.5219\,p$$
$$= 2.609\,p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Similarly, when the steam follows 0.1 of the stroke, 0.1 of a unit of steam will be used per stroke, and the total work of a unit will be

$$W_{0.1} = 10\,p_m = 10 \times 0.3303\,p$$
$$= 3.303\,p. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (j)$$

It will be observed that the coefficient of p, in each of the results, from (a) to (j), is equal to $1 + log_e$ of the measure of expansion. If, therefore, the total work of a unit of steam used without expansion be unity, the gain in work due to expansion in any case, will be expressed by the hyp. log. of the measure of expansion.

This, indeed, might be inferred from (35).

Collecting and tabulating the results just found, the relations will be presented at one view.

TABLE SHOWING THEORETICAL RESULTS OF EXPANSION.

Period of Admission.	Initial Pressure p.	Mean Pressure p_m	Total Work of Unit of Steam.	Gain in Work due to Expansion.	
				Absolute.	Per Cent.
0.1	p.	0 3303 p	3.303 p	2.303 p	230.3
0.2	p.	0.5219 p	2.609 p	1.609 p	160.9
0.3	p.	0.6619 p	2.206 p	1.206 p	120.6
0.4	p.	0.7665 p	1.916 p	0 916 p	91.6
0.5	p.	0.8466 p	1.693 p	0.693 p	69.3
0.6	p.	0.9065 p	1.511 p	0.511 p	51.1
0.7	p.	0.9494 p	1.356 p	0.356 p	35.6
0.8	p.	0.9785 p	1.223 p	0.223 p	22.3
0.9	p.	0.9948 p	1.105 p	0.105 p	10.5
1.0	p.	1.0000 p	1.000 p		

It will be noticed that the gains, indicated in the foregoing table are only theoretical gains, in the total work of the steam, on the steam side of the piston. They do not by any means represent the available gains, in the net or effective work of the engine, which, alone are commercially valuable.

It is not likely that, even in the most favorable cases, the gain in net work has ever reached 100%. The reasons for this will appear later.

64. *Economy in Net Work as Affected by Expansion and Pressure.*—Fig. 18. The net work per stroke, is the total work, less the work performed in overcoming all prejudicial resistances. The latter consists of the tension of the vapor in the condenser, due to the imperfect vacuum, the pressure required to overcome the friction of the unloaded engine, and of the machinery which finally performs the work, and the pressure required to overcome the friction due to the load on the engine and machinery.

In Fig. 18, the work areas due to two pressures, and to two measures of expansion, are represented.

The bottom line is the line of zero pressure, or perfect vacuum line.

a b is the line of prejudicial resistances, and is drawn at a distance above the zero line equal to the sum of the prejudicial resistances per square inch. *c d* is the mean pressure line for the smaller initial pressure, and *c′ d′* is the mean pressure line for the greater initial pressure.

The vertical distance between the mean pressure and prejudi-

cial resistance lines, represents the net or effective pressure, per square inch.

The distance between $c' d'$ and $a b$, being greater than that between $c d$ and $a b$, it follows that the net or effective pressure, and therefore the net work per stroke, is greater in the second case than it is in the first.

It follows from this, that greater economy in net power is secured by high than by low pressures; or that the cost of the net power, in steam, is less as the steam pressure used is greater.

It may be suggested that when the pressure is increased—the measure of expansion being constant—a greater weight of steam will be used per stroke of piston. The answer to this, is found in the fact that, as the pressure is increased, the area of the piston may be diminished in about the same ratio; so that very nearly the same weight of steam will suffice, per stroke of piston, in each case.

With a given pressure, the measure of expansion which will insure the least cost of, or maximum economy in, net power, is that which renders the product of the net pressure, by the distance traversed by the piston, due to the consumption of a unit of steam, a maximum.

If p_m be the mean pressure per square inch, and p_r be the sum of the prejudicial resistances, the net pressure will be $p_m - p_r$. If the unit of steam be a cylindrical volume, having an area of one square inch and a length equal to the stroke of the piston, $= l$, then, when the steam is used without expansion, the net work of the unit of steam, will be that due to a single stroke of the piston and one square inch of its area, and will be expressed by

$$W_n = l\,(p_m - p_r)$$

If the steam be cut off at half-stroke, two strokes of the piston will be made by the unit of steam. But the mean pressure per stroke will, in that case, be greater than half the mean pressure in the first case, and therefore the net work will be greater than

$$2\,l\left(\frac{p_m - p_r}{2}\right) = l\,(p_m - p_r).$$

The steam pressure used, is supposed to be equivalent to that used in practice. If it were very low, the conclusion which has been reached would not be correct.

For example, suppose the initial pressure to be just equal to the sum of the prejudicial resistances, or to p_r; in such a case the piston would not move, and the net work would necessarily be zero. Again, suppose the period of admission to be so short that $p_m = p_r$. In this case, the piston, under the excess of the initial pressure over p_r would start; but when after expansion, the pressure of the steam fell below p_r, the resistance being greater than the pressure of the steam, it would stop. Thus a single stroke of the piston could not be effected, and the net work would again be, practically, zero.

These considerations make it entirely clear, that there must be a point, beyond which expansion, in a given case, cannot be profitably carried; and that, under given conditions, there must be some moderate measure of expansion, which insures the development of net power at a minimum cost, or at maximum economy. The mode of determining this measure of expansion, analytically, which is simple, will be given later.

The principle may, however, be more clearly illustrated, by actual determinations of the net works due to measures of expansion, varying from 1 to 10.

Let the initial pressure $p = 39$ pounds above zero, and the sum of the prejudicial resistance, $p_r = 8$ pounds. Let, also, the area of the piston be one square inch, and the stroke of the piston, unity.

The unit of steam will just fill this cylinder when the piston is at the end of its stroke.

First—Let the cut-off be at 0.1 of the stroke. Then, Art. 62,

$$\begin{array}{rr} p_m = 0.33 \times 39 = & 12.87 \\ p_r = & 8. \\ \hline p_m - p_r = & 4.87. \end{array}$$

Ten strokes of the piston will be effected by the unit of steam, and the net work will be

$$W_n = 10\,(p_m - p_r) = 10 \times 4.87 = 48.70 \text{ foot-pounds.}$$

Second—Let the cut-off be at 0.2 of the stroke.

Then,

$$p_m - p_r = 0.52 \times 39 - 8 = 12.28 \text{ pounds.}$$

Five strokes will be effected by the unit of steam, and the resulting net work will be

$$W_n = 5(p_m - p_r) = 5 \times 12.28 = 61.40 \text{ foot-pounds.}$$

Third—Let the cut-off be at 0.3 of the stroke.
Then,

$$p_m - p_r = 0.66 \times 39 - 8 = 17.74 \text{ pounds.}$$

Three and a third strokes will be effected by the unit of steam. and the resulting net work will be

$$W_n = 3\tfrac{1}{3}(p_m - p_r) = 3\tfrac{1}{3} \times 17.74 = 59.13 \text{ foot-pounds.}$$

Fourth—Let the cut-off be at 0 4 of the stroke.
Then,

$$p_m - p_r = 0.77 \times 39 - 8 = 22.03 \text{ pounds.}$$

Two and one-half strokes will be effected by the unit of steam, and the resulting net work will be

$$W_n = 2\tfrac{1}{2}(p_m - p_r) = 2\tfrac{1}{2} \times 22.03 = 55.08 \text{ foot-pounds.}$$

Fifth—Let the cut-off be at 0.5 of the stroke.
Then,

$$p_m - p_r = 0.85 \times 39 - 8 = 25.15 \text{ pounds.}$$

Two strokes will be effected by the unit of steam, and the resulting net work will be

$$W_n = 2\ (p_m - p_r) = 2 \times 25.15 = 50.30 \text{ foot-pounds.}$$

Sixth—Let the cut-off be at 0.6 of the stroke.
Then,

$$p_m - p_r = 0.906 \times 39 - 8 = 27.33 \text{ foot-pounds.}$$

One and four-sixths strokes will be effected by the unit of steam, and the resulting net work will be

$$W_n = 1\tfrac{2}{3}(p_m - p_r) = 1\tfrac{2}{3} \times 27.33 = 45.55 \text{ foot-pounds.}$$

Seventh—Let the cut-off be at 0.7 of the stroke.
Then,

$$p_m - p_r = 0.95 \times 39 - 8 = 29.05 \text{ pounds.}$$

One and three-sevenths strokes will be effected by the unit of steam, and the resulting net work will be

$$W_n = 1\tfrac{3}{7}(p_m - p_r) = 1\tfrac{3}{7} \times 2.905 = 41.50 \text{ foot-pounds.}$$

Eighth—Let the cut-off be at 0.8 of the stroke.
Then,

$$p_m - p_r = 0.98 \times 39 - 8 = 30.22 \text{ pounds.}$$

One and one-fourth strokes will be effected by the unit of steam, and the resulting net work will be

$$W_n = 1\tfrac{1}{4}(p_m - p_r) = 1\tfrac{1}{4} \times 30.22 = 37.78 \text{ foot-pounds.}$$

Ninth—Let the cut-off be at 0.9 of the stroke.
Then,

$$p_m - p_r = 0.99 \times 39 - 8 = 30.61 \text{ pounds.}$$

One and one-ninth strokes will be effected by the unit of steam, and the resulting net work will be

$$W_n = 1\tfrac{1}{9}(p_m - p_r) = 1\tfrac{1}{9} \times 30.61 = 34.01 \text{ foot-pounds.}$$

Tenth—Let the steam follow full stroke.
Then,

$$p_m - p_r = 1 \times 39 - 8 = 31 \text{ pounds.}$$

A single stroke will be effected by the unit of steam, and the resulting net work will be

$$W_n = 1 \times 31 = 31.00 \text{ foot-pounds.}$$

An algebraic expression for the resulting net work of a unit of steam, when it is used with any measure of expansion, may be constructed as follows:

Employing the notation of Art. 61, we have, when the steam follows a distance a, $\frac{l}{a}$ strokes effected by a unit of steam, and a mean net pressure, during each stroke of $p_m - p_r$.

The resulting net work of the unit of steam will then be expressed by

$$W_n = \frac{l}{a}(p_m - p_r)\,l$$

But (36), substituting $\frac{l}{a}$ for l, in the denominator, gives

$$p_m = \frac{pa}{l}\left(1 + \log_e \frac{l}{a}\right);$$

which, substituted in the value of W_n, gives us

$$W_n = \frac{l}{a}\left[\frac{pa}{l}\left(1 + \log_e \frac{l}{a}\right) - p_r\right] l$$

$$= \left[p\left(1 + \log_e \frac{l}{a}\right) - \frac{p_r l}{a}\right] l. \qquad \text{.............. (37).}$$

Example.—Let $p = 39$; $l = 1$; $a = 0.1$, and $p_r = 8$. Then

$$W_n = 39\,(1 + 2.303) - \frac{8 \times 1}{0.1}$$

$$\doteq 39 \times 3.303 - 80$$

$$= 48.817 \text{ foot-pounds;}$$

which is slightly larger than before, because in that case the last decimal was neglected.

Collecting and tabulating the foregoing results, together with their reciprocals, we can more readily compare the relative values of the net work due to the several measures of expansion.

TABLE OF THE NET WORKS DUE TO DIFFERENT MEASURES OF EXPANSION—WITH THEIR RECIPROCALS.

p. lbs.	p_r lbs.	$\frac{l}{a}$	$\frac{a}{l}$	W_n foot-pounds.	$\frac{1}{W_n}$
39	8	10	0.1	48.70	0.0205
39	8	5	0.2	61.40	0.0163
39	8	$3\frac{1}{3}$	0.3	59.13	0.0169
39	8	$2\frac{1}{2}$	0.4	55.08	0.0181
30	8	2	0.5	50.30	0.0199
39	8	$1\frac{2}{3}$	0.6	45.55	0.0219
39	8	$1\frac{3}{7}$	0.7	41.50	0.0241
39	8	$1\frac{1}{4}$	0.8	37.78	0.0265
39	8	$1\frac{1}{9}$	0.9	34.01	0.0294
39	8	1	1.0	31.00	0.0323

The reciprocals in the last column of this table are measures of nothing; they are a series of numbers which are simply *proportional* to the *costs* of the net power or work due to the corresponding measures of expansion in the third column, or to the corresponding points of cut-off in the fourth column.

This follows, because the numbers of foot-pounds of net work in the fifth column are, in each case, obtained at the cost of a unit of steam. Strictly, in this view of the case, the reciprocals in the last column represent, in each case, the fraction of the unit of steam which performs, under the assumed conditions, a foot-pound of net work. It will be observed that the maximum net work, and the minimum cost at which a given quantity of net work is obtained, correspond to a point of cut-off somewhere between 0.1 and 0.2 of the stroke.

It will also be observed, that the net work and its cost, are very nearly the same, when the cut-off is at some point between

0.4 and 0.5, that they are when the cut-off is at some point between 0.1 and 0.2.

The minimum cost of a given quantity of net work in the table is at 0.2, and is practically 50 per cent. of the cost of the same net work when the steam is used without expansion.

It would appear, therefore, that, without taking account of various modifications yet to be considered, the maximum gain in net work, due to expansion, is about 100 per cent.—under the assumed conditions.

It has been shown in a previous article that, when steam is used without expansion, the cost of a total horse-power, in pounds of steam per hour, is theoretically 35.7 pounds.

In our example, the ratio of total to net work, when the steam is used without expansion, is 39 to 31.

The minimum theoretical cost per net horse-power per hour, therefore, under the assumed conditions, would appear to be

$$\frac{39 \times 35.7}{31 \times 2} = 22.46 \text{ pounds of steam;}$$

or, taking the evaporation at 10 pounds of steam per pound of coal, the theoretical cost, per net horse-power per hour, in coal, would appear to be

$$\frac{22.46}{10} = 2.246 \text{ pounds.}$$

These results, which are purely theoretical, and are subject to various modifications before they can become properly comparable with practical results, may, nevertheless, be roughly compared.

For this purpose, we may take the case of the very efficient three-cylinder compound pumping engines of the Allegheny City, Pa., water-works.

During the test of these engines, in September, 1884, steam was carried at $p = 123$ pounds, and was expanded 17.839 times; and the cost of a net horse-power per hour, in coal, was 1,905 pounds.

64[a]. *The Measure of Expansion which renders W_n a Maximum.**— Eq. (37), may be written

$$W_n = \frac{l}{a}\left(p\,a + p\,a\,log_e \frac{l}{a} - p_r\,l\right).$$

Regarding a as variable, and differentiating, we get

$$d\,W_n = \frac{l}{a}\left[p\,d\,a + p\,a\,d\left(log_e \frac{l}{a}\right) + p\,log_e \frac{l}{a}\,d\,a\right]$$
$$+\left[p\,a + p\,a\,log_e \frac{l}{a} - p_r\,l\right]d\,\frac{l}{a}.$$

Dividing by $d\,a$ and making the first derivative of W_n, with respect to a, equal to zero, we get

$$\frac{d\,W_n}{d\,a} = p\,\frac{l}{a} + p\,\frac{l}{a}\,log_e \frac{l}{a} - \frac{p\,a\,l}{a^2} - \frac{p\,a\,l}{a^2} - \frac{p\,a\,l}{a^2}\,log_e \frac{l}{a}$$
$$+ p_r\left(\frac{l}{a}\right)^2 = 0;$$

or,

$$p\,\frac{l}{a}\left[1 + log_e \frac{l}{a} - 1 - 1 - log_e \frac{l}{a}\right] + p_r\left(\frac{l}{a}\right)^2 = 0;$$

whence

$$p\,\frac{l}{a} = p_r\left(\frac{l}{a}\right)^2.$$

$$\therefore p = p_r\,\frac{l}{a},$$

and

$$\frac{l}{a} = \frac{p}{p_r};$$

or,

$$a = \frac{p_r}{p}\,l. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (37^a)$$

Substituting $\frac{p}{p_r}$ for $\frac{l}{a}$ in the second derivative, the sign will be minus; hence, when steam is cut off at $\frac{p_r}{p}$ of the stroke the net work is a maximum.

*By M. W. Easby (R. P. I., 1886).

Example.—If $p = 39$ lbs. and $p_r = 8$ lbs.

$$a = \frac{p_r}{p}\, l = \frac{8}{39}\, l = 0.205\, l.$$

Eq. (37ᵃ) is interpreted thus: a varies directly as p_r and inversely as p. If p be constant and p_r be made smaller, a becomes smaller; but if, while p is constant, p_r be increased, a will be increased.

If p_r be constant and p be increased, a will be diminished; but if, while p_r is constant, p be diminished, a will be diminished.

But the measure of expansion, $\frac{l}{a}$, varies inversely as a; hence, as p_r is diminished and p increased, the measure of expansion which insures the maximum net work from the steam is increased. Similarly an increase in p_r and a smaller value of p requires a smaller measure of expansion to insure the maximum of net work from the steam.

It is important to remember, however, that these indications are correct only within certain limits; because of a further modification of our expression, for net work, which is necessary before we reach practical conditions.

65. *Graphical Representations.*—Figs. 19 and 19ᵃ. In order to study the effect of expansion upon the quantity and cost of net power, a graphical representation of the net works, in the fifth column of the table of Art. 64, may be constructed as follows:

Lay off on an axis of abscissæ some convenient distance, $A\,B$, Fig. 19, to represent the stroke, and divide this distance into ten equal parts. At each point of division, erect perpendiculars, and on them lay off, according to a convenient scale, a series of ordinates representing the net works of the fifth column of the table. Finally, draw a fair curve through the upper extremities of the ordinates; this curve, of which (37) is the equation, may be designated *the curve of net power, as affected by expansion*—true, of course, only under the assumed conditions of our example.

The curve of net work will cut the axis of X at the origin of coördinates; because if the piston moves through zero distance before the steam is cut off, the net work must be zero.

The curve of cost, Fig. 19[a], may be constructed in a similar manner, as follows: Lay off the stroke $A\ B$ on $A\ X$, divide it into ten equal parts as before, erect perpendiculars at the points of division, and lay off on them according to a convenient scale—larger than that before used—in regular order, from A to B, the values in the last column of the table, and draw a fair curve through the extremities of the ordinates. This curve may be designated ***the curve of cost of net power, as affected by expansion*** —under the assumed conditions of the example.

66. *Clearance and Its Effect.*—Fig. 20. Clearance is the space between the steam valve and the piston, when the latter is at the end of its stroke, which must be filled with steam before the movement of the piston begins. Hitherto it has been assumed that the volume of steam was exactly measured by the space displacement of the piston. This assumption, although the condition cannot be satisfied, has been made for the purpose of avoiding, as long as possible, any confusing complications; and also in order that the effect of clearance, upon the economical development of steam power might, at a later period, be ascertained by a comparison of net works, and their costs, without clearance, with the like results obtained from steam in a cylinder having a definite clearance.

It will be seen that, since clearance requires the expenditure of an increased volume, and weight, of steam for each stroke of the piston, the effect must be to diminish the net work per unit of steam, and therefore to increase its cost; thereby impairing the efficiency and economy of the machine.

In order to ascertain a measure of the effect of clearance, let the rectangle in full lines, Fig. 20, represent the cylinder, without clearance, and the work, per stroke, of the steam, as heretofore; and let the dotted rectangular addition on the left represent the clearance.

Then, when the steam follows full stroke, the volumes of steam required per stroke will be represented by the ***sum*** of the two rectangles when there is clearance, and by the rectangle in full lines when the clearance is zero.

In this case the effect of the clearance is readily determined; the total works of a unit of steam in the two cases being

inversely proportional to the quantities of steam used per stroke, or to the two areas indicated.

When, however, the steam is used expansively, the case becomes more complicated. In this case, the work per stroke may be separated into two parts:

1. The work done before expansion begins.
2. The work done during expansion.

The clearance steam does no work during the period of admission; but during the period of expansion, the expansion of the clearance steam will increase the work beyond what it would be in the absence of the clearance steam.

The two dotted expansion lines inclose a space which represents the effect of the clearance steam, upon both the pressure and work, during the period of expansion. Let c be the clearance, equal to the altitude of the cylinder whose diameter is the same as that of the steam cylinder, and whose volume is equal to the clearance space. Then, using the notation of Articles 61 and 64, we shall have, for the total work per stroke

$$W = p'_m \, l;$$

in which l is the space actually moved through by the piston.

We have from the above

$$p'_m = \frac{W}{l}.$$

Let us now suppose that the piston moves through $l + c$ instead of l. We shall then have, for the work before expansion,

$$w = p\,(a + c),$$

and for the work during expansion,

$$w_1 = p\,(a + c)\,log_e \frac{l + c}{a + c}.$$

The work due to the clearance, which, although not performed, is included in the values of w and w_1, is expressed by

$$w_2 = p\,c.$$

We therefore have, finally,

$$W = w + w_1 - w_2.$$

Substituting for w, w_1 and w_2 their values, factoring and dividing by l, we get

$$p'_m = \frac{W}{l} = \frac{\left[p\,(a+c)\left(1 + log_e.\frac{l+c}{a+c}\right) - p\,c\right]}{l}. \quad \ldots.(38)$$

In this case, however, the volume of steam used is $a + c$, as against a, when the clearance is zero.

In order, therefore, that our results may be comparable, we must multiply (38) by

$$\frac{a}{a+c}.$$

We then get, for the mean total pressure due to the volume a, of steam, used with a clearance c,

$$p_m = p.\frac{a}{a+c}\frac{\left[(a+c)\left(1 + log_e.\frac{l+c}{a+c}\right) - c\right]}{l}. \quad \ldots\ldots..(39)$$

To illustrate the application of (39).

Let $a = 10$; $c = 10$; and $l = 100$.

Then, (39),

$$p_m = p.\frac{10}{10+10}\frac{\left[(10+10)\left(1 + log_e.\frac{100+10}{10+10}\right) - 10\right]}{100}.$$

Now,

$$\frac{100+10}{10+10} = 5.5; \text{ and } log_e.\ 5.5 = 1.705.$$

The above therefore becomes

$$p_m = p \times \frac{1}{2} \times \frac{20 \times 2.705 - 10}{100}$$

$$= 0.2205\,p.$$

Similarly, we may find the values of p_m for $a = 20$, 30, 40, etc., up to 100, or full stroke. These values will be found in the following table:

TABLE OF MEAN TOTAL PRESSURES—BOTH WITH AND WITHOUT CLEARANCE—AND THE LOSSES, PER CENT., DUE TO CLEARANCE.

Cut-off	Mean Pressure — p_m		Loss, per cent., due to Clearance.
a	$c = 0$	$c = 10\%$	
0.1	$0.3303p$	$0.2205\,p$	33.3
0.2	$0.5219p$	$0.3930\,p$	24.7
0.3	$0.6619p$	$0.5286\,p$	20.1
0.4	$0.7665p$	$0.6360\,p$	17.0
0.5	$0.8466p$	$0.7217\,p$	15.0
0.6	$0.9065p$	$0.7879\,p$	13.0
0.7	$0.9494p$	$0.8351\,p$	12.0
0.8	$0.9785p$	$0.8703\,p$	11.0
0.9	$0.9948p$	$0.8955\,p$	10.0
1.0	$1.0000p$	$0.9100\,p$	9.0

Taking, now, the examples of Art. 64, and deducting the losses shown in the last column of the preceding table from the values of p_m, or, which amounts to the same, taking the values of p_m, for a clearance of 10 per cent. in the table, we get, for the work due to each measure of expansion,

$$W_n = (p_m - p_r)\,\frac{l}{a} \dots\dots\dots\dots\dots\dots (40)$$

If
$$\begin{aligned}
a &= 0.1, & W_n &= (0.22 \times 39 - 8) \times 10 = 5.80. \\
a &= 0.2, & W_n &= (0.39 \times 39 - 8) \times 5 = 36.05. \\
a &= 0.3, & W_n &= (0.53 \times 39 - 8) \times 3\tfrac{1}{3} = 42.23. \\
a &= 0.4, & W_n &= (0.64 \times 39 - 8) \times 2\tfrac{1}{2} = 42.35. \\
a &= 0.5, & W_n &= (0.72 \times 39 - 8) \times 2 = 40.16. \\
a &= 0.6, & W_n &= (0.79 \times 39 - 8) \times 1\tfrac{2}{3} = 38.01. \\
a &= 0.7, & W_n &= (0.84 \times 39 - 8) \times 1\tfrac{3}{7} = 35.37. \\
a &= 0.8, & W_n &= (0.87 \times 39 - 8) \times 1\tfrac{1}{4} = 32.41. \\
a &= 0.9, & W_n &= (0.89 \times 39 - 8) \times 1\tfrac{1}{9} = 29.90. \\
a &= 1.0, & W_n &= (0.91 \times 39 - 8) \times 1 = 27.49.
\end{aligned}$$

It will be observed that the maximum value of W_n is obtained when $a = 0.4$; or when the steam follows 0.4 of the stroke.

The maximum apparent gain, in net power, is

$$100 \times \frac{42.35 - 27.49}{27.49} = 54\%.$$

The foregoing results are, of course, only approximations to the truth, when the values of p, p_r, and c are as stated in the example.

For other values, the calculations would be made in the same manner. The clearance in the foregoing example, (10%), is large, In modern engines, the clearance is frequently as small as 2.5 per cent., and is rarely greater than 3.5 per cent.

In order to illustrate still further the application of (39), let us take two cases, of engines actually constructed and now in operation.

1. The Holly Quadruplex Engines of the Buffalo water-works. In these, when operated as non-compound engines, $c = 12.5$, $a = 20$ and $l = 100$.

These values in (39), give

$$p_m = 0.25\,p.$$

If, $a = 30$,

$$p_m = 0.503\,p.$$

2. The Holly Quadruplex Engines of the Troy water-works. In these, which are operated as non-compound engines, $c = 2.6$, $a = 20$ and $l = 100$.

These values, in (39), give

$$p_m = 0.48\,p.$$

If, $a = 30$,

$$p_m = 0.62\,p.$$

The four values of p_m, representing absolute work, give the following ratios:

1. $0.25\,p \;:\; 0.25\,p = 1.$
2. $0.50\,p \;:\; 0.25\,p = 2.$
3. $0.48\,p \;:\; 0.25\,p = 1.92$
4. $0.62\,p \;:\; 0.50\,p = 1.24.$

If the values of p_r be introduced, as in (40), the differences in the resulting values of W_n would be still more marked.

Our object thus far has been, not so much to present precise results, as to show, in a general way, the extent to which the theoretical gain due to the expansion of steam is modified by the conditions under which power is produced in practice; and to emphasize the importance of reducing *clearance*, and *prejudicial resistances*, to the smallest practicable limits; in order that the ratios of net to total powers, for given measures of expansion, may have the largest practicable values.

Eq. 39 shows that, other things being the same, the value of p_m is increased by diminishing c; while Eq. 40 shows that W_n is increased as p_m is increased and as p_r is diminished.

W_n, it will be remembered, represents the net, or commercially valuable power. It therefore follows, that the larger the value of the work or power W_n, for a given expenditure of steam, the more economically the power will be produced.

67. *Efficiency of the Engine.*—By efficiency is meant, either the relation between the power usefully applied by, and the power expended upon, the engine, or the relation between the useful work performed and the work due to the heat expended in generating the steam. In the first case, the result depends upon the working condition of the mechanism, and expresses its efficiency as a receiver and transmitter of power; while in the second case it depends, not alone upon the working conditions of the mechanism, but upon those details of design and arrangement which are intended to insure the most effective application of the steam, by the utilization of its heat, as well.

If

R = the useful load, or net power of an engine,
R_0 = the friction of the unloaded engine,
f = a co-efficient of R, expressing the relation of the friction due to the useful load, to the useful load itself, and
E_m = the efficiency of the mechanism,

the expression for the efficiency of the engine, as a receiver and transmitter of power, will be

$$E_m = \frac{R}{R(1+f) + R_0}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

f varies from 0.03 to 0.14; and R_0 is the power due to a pressure of from 1 to $2\frac{1}{2}$ pounds per square inch of piston.

If $R = 85$ horse-power, $f = 0.08$ and $R_0 = 5$ horse-power.

$$E_m = \frac{85}{85 \times 1.08 + 5} = 0.878.$$

If the engine and boiler be regarded as the engine, and

W = the work of a pound of steam,
W_h = the equivalent, in work, of the heat expended in generating a pound of steam,
E_e = the efficiency of the engine,

we have

$$E_e = \frac{W}{W_h}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Let us now regard the engine as simply a steam engine, and deduce an expression for efficiency, which shall be the ratio of the net or useful work performed, to the total theoretical work of the steam when used in a perfect engine—or in an engine where there are no prejudicial resistances, and in which the clearance is zero.

Let

$W_p =$ the work of a unit of steam in a perfect engine,

$W_u =$ the net or useful work of a unit of steam, and

$E_s =$ the efficiency of the steam engine.

Then

$$E_s = \frac{W_u}{W_p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Now, to find an expression for W_p, we have, Eq. (35), for the work per stroke of piston,

$$w = p\,a\left(1 + log_e.\frac{l}{a}\right);$$

but $\frac{l}{a}$ strokes will be made by the unit of steam.

$$\therefore W_p = \frac{w\,l}{a} = \frac{p\,a\,l}{a}\left(1 + log_e.\frac{l}{a}\right)$$

$$= p\,l\left(1 + log_e.\frac{l}{a}\right);$$

or, making $l =$ unity,

$$W_p = p\left(1 + log_e.\frac{l}{a}\right). \quad \ldots\ldots\ldots (44)$$

Again, in Eq. (39) we have an expression for the mean pressure, or work—stroke = unity—for a volume a of steam, when the clearance $= c$.

Subtracting p_r from (39), for the combined effect of imperfect vacuum and all other prejudicial resistances, we have for the net or useful mean pressure per stroke,

$$p_m = p.\frac{a}{a+c}.\frac{\left[(a+c)\left(1 + log_e.\frac{l+c}{a+c}\right) - c\right]}{l} - p_r.$$

Multiplying this mean net pressure by the stroke l, we get for the net work per stroke, or of the volume a, of steam,

$$p_m l = p.\frac{a}{a+c}.\left[(a+c)\left(1 + log_e.\frac{l+c}{a+c}\right) - c\right] - p_r l.$$

The unit of steam will make $\frac{l}{a}$ strokes of the piston. Multiplying the above expression by $\frac{l}{a}$, and making l = unity, we get finally,

$$W_u = \frac{p}{a+c} \cdot \left[(a+c)\left(1 + \log_e \frac{l+c}{a+c}\right) - c\right] - \frac{p_r}{a}. \quad \ldots\ldots(45)$$

Substituting now the values of W_u and W_p, in (44) and (45), in (43), we get

$$E_s = \frac{\frac{p}{a+c} \cdot \left[(a+c)\left(1 + \log_e \frac{l+c}{a+c}\right) - c\right] - \frac{p_r}{a}}{p\left(1 + \log_e \frac{l}{a}\right)}. \quad \ldots\ldots\ldots(46)$$

Examples.—Let $p = 80$; $l = 1.0$; $a = 0.2$; $c = 0.125$, and $p_r = 2 + 2 + 3 = 7$ pounds.

Then,

$$W_u = \frac{80}{0.2 + 0.125}\left[0.325\left(1 + \log_e \frac{1.125}{0.325}\right) - 0.125\right] - \frac{7}{0.2}.$$

$$= 246.1\ [0.325\ (1 + \log_e 3.46) - 0.125] - \frac{7}{0.2}.$$

$$= 246.1 \times 0.603 - 35 = 113.4;$$

and

$$W_p = 80\ (1 + \log_e 5)$$
$$= 80\ (1 + 1.609) = 208.72.$$

Then,

$$E_s = \frac{W_u}{W_p} = \frac{113.4}{208.72} = 0.545.$$

This example involves the conditions of the Holly quadruplex engine of the Buffalo water works, operated as a non-compound engine, during a trial. In this case the small efficiency results from the excessively large clearance.

Second—Let $p = 90$; $l = 1.0$; $a = 0.3$; $c = 0.026$, and $p_r = 2 + 2 + 3 = 7$ pounds, as before.

Then,

$$W_u = \frac{90}{0.3 \times 0.026}\left[0.326\left(1 + log_e.\frac{1.026}{0.326}\right) - 0.026\right] - \frac{7}{0.3}$$

$$= 276.1\ [0.326\ (1 + log_e.\ 3.15) - 0.026] - \frac{7}{3.0}$$

$$= 271.1 \times 0.675 - 23.33 = 163.037;$$

and

$$W_p = 90\ (1 + log_e.\ 3.33)$$

$$= 90 \times 2.203 = 198.18.$$

Then,

$$E_s = \frac{W_u}{W_p} = \frac{163.037}{198.18} = 0.822.$$

The second example involves the conditions of the Holly quadruplex engines of the Troy water-works.

We may add, that the efficiencies which we have found are almost exactly proportional to the duties shown by the two engines. It is but just, however, to state that the Buffalo engine was designed to run, and is habitually run, as a compound engine; and that its duty is, ordinarily, about 80 millions.

68. *Efficiency of the Engine, regarded as a Heat-Engine.*—It has been shown, Art. 50, that the work of one pound of steam, when it is used in an hour, and without expansion, is 0.028 of a horse-power; and, therefore, that the evaporation required, in order to maintain an absolute horse-power, must be $\frac{1}{0.028} = 35.7$ pounds of water per hour.

It was also shown that the ratio of the total work of a pound of steam, expressed in heat units, to the total heat of the steam, above 39.°1 Fahr., or the efficiency of the steam, when used as stated, is 0.063.

Now, taking our expressions of Art. 67, we may convert them into expressions for the efficiency of the engine, regarded as a heat-engine, as follows:

The efficiency of what has been designated the perfect engine when the steam is used expansively, is evidently

$$0.063\left(1 + log_e.\frac{l}{a}\right)$$

Now E_u, in Eq. (46), is the ratio of the net or useful work of an engine, under the conditions of practice, to the theoreti-

cal work of the perfect engine, with the same period of admission.

It follows, then, that if Eq. (46) be multiplied by the above expression, the result will be the ratio of the net or useful work to the total work due to the total heat of the steam, less 39.°1, the assumed initial temperature of the water.

If, then, E_s^h represents the efficiency of the engine, regarded as a heat-engine, we shall have

$$E_s^h = 0.063\left(1 + log_e.\frac{l}{a}\right)E_s. \quad \ldots\ldots\ldots(47)$$

Example.—Taking the first example of Art. 67, in which $a = 2$, $\frac{l}{a} = 5$, and $E_s = 0.545$, and substituting in (47), we get

$$E_s^h = 0.063 \times 2.609 \times 0.545$$
$$= 0.08958.$$

Example 2.—Taking the second example of Art. 67, in which $a = 0.3$, $\frac{l}{a} = 3\frac{1}{3}$ and $E_s = 0.822$, and substituting in (47), we get

$$E_s^h = 0.063\,(1 + log_e.\,3\tfrac{1}{3}).\,0.822$$
$$= 0.063 \times 2.203 \times 0.822$$
$$= 0.1141.$$

By performing the operations indicated in (47), substituting for E_s, its value is (46), we get

$$E_s^h = \frac{0.063}{a+c}\left[(a+c)\left(1 + log_e.\frac{l+c}{a+c}\right) - c\right] - \frac{0.063\,p_r}{a\,p}. \quad \ldots.(48)$$

Making the proper substitutions in (48), the theoretical value of E_s^h may be found in any case.

69. *Pounds of Steam Required per net Horse-power per Hour.*—The weights of steam required to maintain a total theoretical horse-power in our perfect steam engine, using steam without expansion, and to maintain a net horse-power in an actual engine, will evidently be inversely proportional to the efficiences of the engines, as heat engines.

Let W_s = the weight, in pounds, of the steam required to maintain a net horse-power for one hour.

Then

$$35.7 : W_s :: E_s^h : 0.063;$$

whence

$$W_s = 35.7 \times \frac{0.063}{E_s^h};$$

or, substituting for E_s^h its value in (47), we get

$$W_s = \frac{35.7 \times 0.063}{0.063\left(1 + log_e.\ \frac{l}{a}\right) E_s}$$

$$= \frac{35.7}{E_s\left(1 + log_e.\ \frac{l}{a}\right)}. \quad \ldots\ldots\ldots\ldots\ldots (49)$$

Example 1.—If $E_s = 0.545$, $a = 0.2$ and $\frac{l}{a} = 5$, (49) gives

$$W_s = \frac{35.7}{0.545 \times 2.609} = 25.16 \text{ pounds.}$$

Example 2.—If $E_s = 0.822$, $a = 0.3$ and $\frac{l}{a} = 3\frac{1}{3}$, (49) gives

$$W_s = \frac{35.7}{0.822 \times 2.203} = 20 \text{ pounds.}$$

From (46) and (49), it appears that, with higher pressures and larger measures of expansion—other things being the same—the weights of steam required to maintain a net horse-power per hour, would be still less than those indicated in the foregoing example:

A general formula for W_s is obtained by substituting for E_s in (49) its value in (46); thus

$$W_s = \frac{35.7\,p}{\frac{p}{a+e}\left[(a+c)\left(1 + log_e.\ \frac{l+c}{a+c}\right) - c\right] - \frac{p_r}{a}}$$

$$= \frac{35.7}{\frac{1}{a+c}\left[(a+c)\left(1 + log_e.\ \frac{l+c}{a+c}\right) - c\right] - \frac{p_r}{ap}}. \quad \ldots (50)$$

In order to show more clearly the relations between W_s and p and a, the values of W_s have been calculated for various values of a, and for $p = 30$, $p = 45$, and $p = 90$ pounds; c being 0.026

and $p_r = 7$ pounds in each case. The resulting values of W_s will be found in the following table;

a.	W_s in Pounds.		
	$p = 30$.	$p = 45$.	$p = 90$.
0.02	.....	∞	∞
0.04	.....	∞	33.75
0.06	.....	421.70	22.73
0.08	.....	.....	18.67
0.10	166.56	32.29	17.88
0.12	97.49	27.67	17.58
0.15	42.16	24.92	17.62
0.17	37.49	24.21	17.87
0.20	35.57	23.66	18.23
0.25	31.02	23.70	19.18
0.30	30.30	24.21	20.16
0.40	31.21	26.12	22.42
0.50	32.98	28.33	24.77
0.60	35.42	30.89	27.33
0.70	38.54	33.89	30.16
0.80	41.67	36.89	33.02
0.90	45.94	40.78	36.55
1.00	50.89	44.79	40.22

These results are instructive, as well as interesting. They show:

1. That, other things being the same, the cost of a net horse-power, in steam, diminishes as the pressure of the steam increases.

2. That the measure of expansion which insures the maintenance of a net horse-power at minimum cost, increases as the pressure increases.*

3. That the cost of maintaining a net horse-power increases very rapidly as the measure of expansion is increased, beyond that which insures the minimum cost.

4. That, as the measure of expansion is diminished, below that which insures the minimum cost, the cost increases; but at a much smaller rate than it increases when the measure of expansion is increased.

It follows, then, that in case the point of cut-off which insures the minimum cost of net power, is theoretically at 0.2 of the stroke, it may be true that, under practical conditions, the same net power may be maintained at less actual cost, with the cut-off at nearly 0.4; because, in that case, the power would be ob-

*See appendix.

tained from a smaller cylinder, which would cost less, would occupy less space, and would radiate less heat.

It should be remarked in this connection, that in actual practice, the theoretical results which we have found are subject to very considerable modification, in consequence of unavoidable losses due to condensation in the cylinder.

These losses, which cannot be precisely estimated, will be referred to, and exemplified, later.

It is recommended that curves be constructed, to a scale, with the several values of a as abscissæ, and with the different values of W_s as ordinates.

The careful and intelligent study of these curves will aid, materially, in forming correct conclusions in regard to the conditions which insure economy in the maintenance of steam power.

VII. The Compound Engine.

70. *Description of the Compound Engine.*—The compound engine has two or three cylinders. If there be two cylinders, the first or smaller cylinder is designated the high-pressure cylinder, and the second, or larger cylinder, the low-pressure cylinder.

If there be three cylinders, the smaller is designated the high-pressure cylinder, and the two larger, the low-pressure cylinders.

In the case of the two-cylinder compound engine, the steam, after doing its work in the high-pressure cylinder, is exhausted into the low-pressure cylinder, where it performs additional work; after which it is exhausted into the condenser.

Where there are three cylinders, the steam, after completing its work in the high-pressure cylinder, is exhausted into a receiver. From the receiver the steam passes to the low-pressure cylinders, and, after doing additional work there, is finally exhausted into the condenser.

In some cases, where steam of very high pressure is used, the steam passes consecutively to, and is used in, the first, second and third cylinders, and thence passes to the condenser.*

71. *Operation of the Two-Cylinder Compound Engine.*—In order to explain the operation of the two-cylinder compound engine, and the total effect of the steam therein, let us take the general arrangement shown in Fig. 21, in which one cylinder is placed behind the other, and in which $a\ b\ c\ d$ is the high-pressure

* Triple expansion engines.

cylinder, and $A\ B\ C\ D$ the low-pressure cylinder—the axes of the two cylinders lying in the same line.

In the diagram, the pistons are represented as moving from right to left, and as being at about half-stroke.

On the right of the high-pressure cylinder, steam of boiler pressure is acting.

On the left of the high-pressure piston, its motion is resisted by the steam, which now occupies about one-half of the capacity of the high-pressure cylinder and about one-half of the capacity of the low-pressure cylinder; but which originally, or during the previous stroke, filled the high-pressure cylinder at the instant that the pistons completed the preceding stroke from left to right.

As the volume of the exhaust steam of the previous stroke is now greater than it was at the instant of the completion of that stroke, its tension is less than that of the boiler steam, and there is, therefore, a resultant moving force, acting from right to left on the high-pressure piston. On the right of the low-pressure piston, the tension of the steam is the same as it is on the left of the high-pressure piston; but since the low-pressure piston is from three to four times as large as the high-pressure piston, and as we assume a perfect vacuum on the left of the low-pressure piston, there is an effective force acting upon the low-pressure piston, from right to left, which is from three to four times the resistance on the left of the high-pressure piston.

As the pistons continue their motion toward the left, more and more of the steam passes from the left-hand end of the high-pressure cylinder to the right-hand end of the low-pressure cylinder; its volume becomes greater and greater, and its tension less, until, at the end of the stroke, the high-pressure volume of steam will have been wholly transferred to the low-pressure cylinder.

Thus, it will be seen, the effective force acting upon the high-pressure piston varies from zero, at the beginning of the stroke, to two-thirds or three-fourths of the pressure on the right-hand side, or the pressure due to the boiler steam, at the end of the stroke. On the low-pressure piston the pressure, on the contrary, gradually diminishes, from the beginning to the end of the stroke.

The combined effect of the action of the steam on both pistons is a resultant effective pressure, which is less at the begin-

ning, and greater at the end of the stroke, than it would be in case the same steam was used in the low-pressure cylinder, with the same measure of expansion.

Assuming the steam to be used without expansion, in the high-pressure cylinder, the measure of expansion is, obviously, the ratio of the volume of the low-pressure cylinder to that of the high-pressure cylinder.

Let us now trace the component and resultant total pressures on the two pistons during a complete stroke of both.

For this purpose let the stroke be separated into ten equal parts.

Let $V =$ volume of the high-pressure cylinder.
$3V =$ volume of the low-pressure cylinder.
$3 =$ measure of expansion.
$P =$ the total steam pressure on the steam side of the high-pressure piston.
$P_r =$ resultant pressure on both pistons.

Then, neglecting the volume of steam in the passages, and taking the pistons at the beginning of a stroke from right to left, we have the following component pressures:

On the right of the *h. p. p.*,	$P.$
" " left " " "	$-P.$
" " right " " *l. p. p.*,	$3P.$
$\therefore \quad P_r = \ldots\ldots\ldots\ldots$	$3P.$

Second—Take the pistons at 0.1 of their stroke.

The steam which previously filled the *h. p. cyl.*, now fills nine-tenths of the *h. p. cyl.* and one-tenth of the *l. p. cyl.*

The total volume of the exhaust from the *h. p. cyl.* is now, therefore,

$$0.9\,V + 0.1 \times 3\,V = 1.2\,V;$$

and its tension is $\frac{10}{12}$ of its original tension.

In this position, then, we have the following component pressures:

On the right of the *h. p. p.*,	$P.$
" " left " " "	$-\frac{10}{12}P.$
" " right " " *l. p. p.*,	$3 \times \frac{10}{12}P.$
$\therefore \quad P_r = \frac{32}{12}P =$	$2.67\,P.$

Third—Take the pistons at 0.2 of their stroke.

The volume of the exhaust steam from the *h. p. cyl.* is now

$$0.8\,V + 0.2 \times 3\,V = 1.4\,V;$$

and its tension is $\frac{10}{14}$ of its original tension.

The component pressures will therefore be as follows:

On the right of the *h. p. p.*,	P.
" " left " " "	$-\frac{10}{14}P$.
" " right " " *l. p. p.*,	$3 \times \frac{10}{14}P$.
$\therefore \quad P_r = \frac{34}{14}P =$	$2.43\,P$.

Fourth—Take the pistons at 0.3 of their stroke.

The volume of the exhaust from the *h. p. cyl.* is now

$$0.7\,V + 0.3 \times 3\,V = 1.6\,V;$$

and its tension is $\frac{10}{16}$ of its original tension.

The component pressures will therefore be as follows:

On the right of the *h. p. p.*,	P.
" " left " " "	$-\frac{10}{16}P$.
" " right " " *l. p. p.*,	$3 \times \frac{10}{16}P$.
$\therefore \quad P_r = \frac{36}{16}P =$	$2.25\,P$.

Fifth—Take the pistons at 0.4 of their stroke.

The volume of the exhaust from the *h. p. cyl.* is now

$$0.6\,V + 0.4 \times 3\,V = 1.8\,V;$$

and its tension is $\frac{10}{18}$ of its original tension.

The component pressures will therefore be as follows:

On the right of the *h. p. p.*,	P.
" " left " " "	$-\frac{10}{18}P$.
" " right " " *l. p. p.*,	$3 \times \frac{10}{18}P$.
$\therefore \quad P_r = \frac{38}{18}P =$	$2.11\,P$.

Sixth—Take the pistons at 0.5 of their stroke.

The volume of the exhaust from the *h. p. cyl.* will now be

$$0.5\,V + 0.5 \times 3\,V = 2.\,V;$$

and its elastic force is $\frac{10}{20}$ of its original elastic force.

The component pressures are then as follows:

On the right of the *h. p. p.*,	P.
" " left " " "	$-\frac{10}{20}P$.
" " right " " *l. p. p.*,	$3 \times \frac{10}{20}P$.
$\therefore \quad P_r = \frac{40}{20}P =$	$2\,P$.

In a similar manner we might determine the resultant pressures at each of the remaining points of the stroke.

This is unnecessary, however, as we have discovered the laws according to which volume and tension of the exhaust steam, and the resultant pressure vary, and may therefore write them at once, in each case.

The following table contains all of the values for each of the eleven points of the stroke.

TABLE SHOWING THE VOLUME AND TENSION OF THE HIGH-PRESSURE, EXHAUST, AND THE RESULTANT PRESSURE ON THE PISTONS, AT EACH OF 11 EQUIDISTANT POINTS OF A STROKE OF THE PISTON OF A TWO-CYLINDER COMPOUND ENGINE.

No.	a	Volume of Exhaust Steam.	Tension of Exhaust Steam.	P_r	
1.	0.0	V	1	$3\ P$	$3.00P$
2.	0.1	$1.2V$	$\frac{10}{12}$	$\frac{32}{12}P$	$2.67P$
3.	0.2	$1.4V$	$\frac{10}{14}$	$\frac{34}{14}P$	$2.43P$
4.	0.3	$1.6V$	$\frac{10}{16}$	$\frac{36}{16}P$	$2.25P$
5.	0.4	$1.8V$	$\frac{10}{18}$	$\frac{38}{18}P$	$2.11P$
6.	0.5	$2.0V$	$\frac{10}{20}$	$\frac{40}{20}P$	$2.00P$
7.	0.6	$2.2V$	$\frac{10}{22}$	$\frac{42}{22}P$	$1.91P$
8.	0.7	$2.4V$	$\frac{10}{24}$	$\frac{44}{24}P$	$1.83P$
9.	0.8	$2.6V$	$\frac{10}{26}$	$\frac{46}{26}P$	$1.77P$
10.	0.9	$2.8V$	$\frac{10}{28}$	$\frac{48}{28}P$	$1.71P$
11.	1.0	$3.0V$	$\frac{10}{30}$	$\frac{50}{30}P$	$1.67P$
Sum of terms less half sum of extremes.					$21.01P$

The mean resultant pressure on both pistons, P_m, will be equal to the sum of all the separate values of P_r less half the sum of the first and last values of P_r, divided by the number of values of P_r less one; or

$$P_m = \frac{21.01\,P}{11 - 1} = 2.101\,P$$

$$= 0.7 \times 3\,P.$$

This is the mean resultant pressure on both pistons, reduced to the large, or *l. p.* piston.

The steam is expanded three times; because, during each stroke, a volume V of steam occupying the *h. p. cyl.*, is transferred to the *l. p. cyl.*, where its volume becomes $3\,V$.

Let us suppose, now, that the same steam which has been consumed during a stroke of both the pistons of the compound engine be used in *l. p. cyl.* alone.

This steam will fill one-third of the *l.p. cyl.*, and will then be expanded three times, when it will entirely fill the *l.p. cyl.* The initial pressure on the entire piston of the *l.p. cyl.* will be $3P$, and the terminal pressure, P.

The mean total pressure during the stroke will be, (36),

$$P_m = \frac{3P(1 + log_e.3)}{3}$$

$$= \frac{3P(1 + 1.10)}{3}$$

$$= 0.7 \times 3P,$$

as before.

It thus appears that, under the conditions assumed, at least, there is no theoretical gain in efficiency in the compound engine; the total work of the steam being precisely the same, in the two-cylinder compound engine, as in the simple engine whose cylinder is equal to the *l.p. cyl.* of the compound engine, and in which the same steam is used with the same measure of expansion.

Fig. 22 is a graphical representation of the net work in the *h. p. cyl.*, in the *l.p. cyl.*, and in both cylinders; also, the equivalent work of the same steam in a simple engine, the steam being used with the same measure of expansion.

f b' e is the work of the steam on the *h.p.* piston.

a f b' b represents the work of the steam on the *l.p.* piston.

a b e f represents the total work of the steam during a single stroke, on both pistons, and

a c d e f represents the work of the same steam, used with the same measure of expansion, in the *l p. cyl.* This diagram should be carefully drawn to a scale.

It will be observed that, in the case of the compound engine, the excess of pressure, during the early part of the stroke, above the mean pressure, is less than it is in the case of the simple engine. The same may be said of the deficiency of pressure during the latter part of the stroke.

It follows, therefore, that the action of the compound engine is more nearly uniform than that of the simple engine, and that for this reason, it requires a smaller regulator, or fly-wheel, than the simple engine.

The work of the fly-wheel, in each case, is represented by the shaded portions of Figs. 23 and 24.

This constitutes a mechanical advantage which the compound engine has over the simple engine. The compound engine has a further mechanical advantage, in that its crank-pin and journals are less liable to heat than those of the simple engine, and in that there is less back-lash, and consequently less shock among the parts of the machinery.

The Worthington, Knowles, Davidson and other pumping-engines are notable examples of the type of two-cylinder compound engines which we have been considering, in which the fly-wheel is wholly dispensed with.

Again, higher steam pressure and larger measures of expansion may be employed with the compound engine, and for the reasons already enumerated, both of which conditions, it will be remembered, are favorable to an economical performance.

72. *Operation of the Compound Engine when Steam is Used Expansively in the High-Pressure Cylinder.*—

Let us suppose that the admission of steam to the *h. p. cyl.* is suppressed when the pistons are at half-stroke.

Let $2V$ = volume of the *h. p. cyl.*,
$6V$ = " " *l. p. cyl.*
P = total pressure on the steam side of the *h. p.* piston.
P_r = resultant pressure on both pistons, at any point of the stroke.
P_m = mean resultant pressure on both pistons, during an entire stroke.

In this case the steam will be expanded six times—twice in the *h. p. cyl.* and three times in its passage from the *h. p. cyl.* to the *l. p. cyl.*

Then,

1. When the pistons are just on the point of beginning a stroke, from right to left, the component pressures will be as follows:

On the right of the *h. p. p.*,	$P.$
" " left " " "	$-\frac{1}{2}P.$
" " right " " *l. p. p.*,	$3 \times \frac{1}{2}P.$
$\therefore P_r =$	$2P.$

$(2P)$

2. When the pistons are at 0.1 of their stroke, the volume of the *h. p.* exhaust will be

$$0.9 \times 2V + 0.1 \times 6V = 2.4V;$$

and its tension will be $\frac{10}{24}$ of the initial tension.

The component pressures will therefore be

On the right of the *h. p. p.*,	$P.$
" " left " " "	$-\frac{10}{24}P.$
" " right " " *l. p. p.*,	$3 \times \frac{10}{24}P.$
$\therefore P_r = \frac{44}{24}P =$	$1.83\,P.$

3. When the pistons are at 0.2 of their stroke, the volume of the *h. p.* exhaust will be

$$0.8 \times 2V + 0.2 \times 6V = 2.8V;$$

and its tension will be $\frac{10}{28}$ of its initial tension.

The component pressures will therefore be

On the right of the *h. p. p.*,	$P.$
" " left " " "	$-\frac{10}{28}P.$
" " right " " *l. p. p.*,	$3 \times \frac{10}{28}P.$
$\therefore P_r = \frac{48}{28}P =$	$1.77\,P.$

4. When the pistons are at 0.3 of their stroke, the volume of the *h. p.* exhaust will be

$$0.7 \times 2V + 0.3 \times 6V = 3.2V;$$

and its tension will be $\frac{10}{32}$ of its initial tension.

The component pressures will therefore be

On the right of the *h. p p.*,	$P.$
" " left " " "	$-\frac{10}{32}P.$
" " right " " *l. p. p.*,	$3 \times \frac{10}{32}P.$
$\therefore P_r = \frac{52}{32}P =$	$1.62\,P.$

5. When the pistons are at 0.4 of their stroke, the volume of the *h. p.* exhaust will be

$$0.6 \times 2V + 0.4 \times 6V = 3.6V;$$

and its tension will be $\frac{10}{36}$ of its initial tension.

The component pressures will therefore be

On the right of the *h. p. p.*,	$P.$
" " left " " "	$-\frac{10}{36}P.$
" " right " " *l. p. p.*,	$3 \times \frac{10}{36}P.$
$\therefore P_r = \frac{56}{36}P =$	$1.55\,P.$

6. When the pistons are at 0.5 of their stroke, the volume of the *h. p.* exhaust will be

$$0.5 \times 2V + 0.5 \times 6V = 4.0V;$$

and its tension will be $\frac{10}{40}$ of its initial tension.

The component pressures will therefore be

On the right of the *h. p. p.*,	P.
" " left " " "	$-\frac{10}{40}P$.
" " right " " *l. p. p.*,	$3 \times \frac{10}{40}P$.
$\therefore P_r = \frac{60}{40}P =$	$1.50P$.

7. When the pistons are at 0.6 of their stroke, the volume of the *h. p.* exhaust will be.

$$0.4 \times 2V + 0.6 \times 6V = 4.4V;$$

and its tension will be $\frac{10}{44}$ of its initial tension.

On the right, or steam side, of the *h. p. p.* a volume V of steam has now expanded to $\frac{6}{5}V = 1.2V$; and its tension is now only $\frac{10}{12}$ of its initial tension.

The component pressures will therefore be

On the right of the *h. p. p.*,	$\frac{10}{12}P$.
" " left " " "	$-\frac{10}{44}P$.
" " right " " *l. p. p.*,	$3 \times \frac{10}{44}P$.
$\therefore P_r = \frac{85}{66}P =$	$1.29P$.

8. When the pistons are at 0.7 of their stroke, the volume of the *h. p.* exhaust will be

$$0.3 \times 2V + 0.7 \times 6V = 4.8V;$$

and its tension will be $\frac{10}{48}$ of its initial tension.

On the right, or steam side, of the piston, the volume V of steam has now expanded to $\frac{7}{5}V = 1.4V$; and its tension is reduced to $\frac{10}{14}$ of its initial tension.

The component pressures will therefore be

On the right of the *h. p. p.*,	$\frac{10}{14}P$.
" " left " " "	$-\frac{10}{48}P$.
" " right " " *l. p. p.*,	$3 \times \frac{10}{48}P$.
$\therefore P_r = \frac{95}{84}P =$	$1.13P$.

Having now discovered the law according to which the volumes, tensions and component pressures change, we may at once write the component pressures for each of the three

remaining positions of the pistons; thus, when the pistons are at 0.8 of their stroke the pressures will be,

$$
\begin{array}{lr}
\text{On the right of the } h.\,p.\,p., & \tfrac{10}{16}\,P. \\
\text{“ “ left “ “ “} & -\tfrac{10}{52}\,P. \\
\text{“ “ right “ “ } l.\,p.\,p., & 3 \times \tfrac{10}{52}\,P. \\
\hline
\therefore P_r = \tfrac{210}{208}\,P = & 1.01\,P.
\end{array}
$$

When the pistons are at 0.9 of the stroke we have,

$$
\begin{array}{lr}
\text{On the right of the } h.\,p.\,p., & \tfrac{10}{18}\,P. \\
\text{“ “ left “ “ “} & -\tfrac{10}{56}\,P. \\
\text{“ “ right “ “ } l.\,p.\,p., & 3 \times \tfrac{10}{56}\,P. \\
\hline
\therefore P_r = \tfrac{230}{252}\,P = & 0.91\,P.
\end{array}
$$

At the end of the stroke the pressures will be,

$$
\begin{array}{lr}
\text{On the right of the } h.\,p.\,p., & \tfrac{10}{20}\,P. \\
\text{“ “ left “ “ “} & -\tfrac{10}{60}\,P. \\
\text{“ “ right “ “ } l.\,p.\,p., & 3 \times \tfrac{10}{60}\,P. \\
\hline
\therefore P_r = \tfrac{5}{6}\,P = & 0.83\,P.
\end{array}
$$

For convenience of reference, the results just found are collected and arranged in the following table:

TABLE SHOWING THE VOLUMES AND TENSIONS OF THE STEAM IN THE CYLINDERS, AND THE COMPONENT AND RESULTANT PRESSURES ON THE PISTONS, OF A TWO-CYLINDER COMPOUND ENGINE, THE STEAM BEING EXPANDED TWICE IN THE *h. p. cyl.*

No.	Positions of Pistons a	*H. P. Cyl.*		*L. P. Cyl.*		P_r.
		Vol.	Tension.	Vol.	Tension.	
1.	0.0	0.0	1	$2.0V$	$\frac{10}{20}$	$2.P = 2.00P$
2.	0.1	$0.2V$	1	$2.4V$	$\frac{10}{24}$	$\frac{44}{24}P = 1.83P$
3.	0.2	$0.4V$	1	$2.8V$	$\frac{10}{28}$	$\frac{48}{28}P = 1.77P$
4.	0.3	$0.6V$	1	$3.2V$	$\frac{10}{32}$	$\frac{52}{32}P = 1.62P$
5.	0.4	$0.8V$	1	$3.6V$	$\frac{10}{36}$	$\frac{56}{36}P = 1.55P$
6.	0.5	$1.0V$	1	$4.0V$	$\frac{10}{40}$	$\frac{60}{40}P = 1.50P$
7.	0.6	$1.2V$	$\frac{10}{12}$	$4.4V$	$\frac{10}{44}$	$\frac{85}{66}P = 1.29P$
8.	0.7	$1.4V$	$\frac{10}{14}$	$4.8V$	$\frac{10}{48}$	$\frac{95}{84}P = 1.13P$
9.	0.8	$1.6V$	$\frac{10}{16}$	$5.2V$	$\frac{10}{52}$	$\frac{210}{208}P = 1.01P$
10.	0.9	$1.8V$	$\frac{10}{18}$	$5.6V$	$\frac{10}{56}$	$\frac{230}{252}P = 0.91P$
11.	1.0	$2.0V$	$\frac{10}{20}$	$6.0V$	$\frac{10}{60}$	$\frac{5}{6}P = 0.83P$
Sum of values of $P_r - \frac{1}{2}$ sum of extreme values.........						$14.025P$.

$$\therefore P_m = \frac{14.025\,P}{11-1} = 1.4025\,P$$

$$3P \times 0.468.$$

Thus it appears that the mean resultant pressure on both pistons, during a stroke, is equal to the initial pressure on the *l. p. p.* which would result from the use of the volume V of steam, in the *l. p. cyl.*, and expanding it six times, multiplied by 0.468.

This would be equivalent to using the same steam in the *l. p. cyl.*, with the same measure of expansion; in other words, it would be equivalent to using the same steam, with the same measure of expansion, in a simple engine.

In that case we should have, (36),

$$P_m = \frac{3\,P(1 + log_e.6)}{6}$$

$$= \frac{3\,P\,(1 + 1.792)}{6}$$

$$= 3\,P \times 0.465,$$

as before.

In a similar manner it may be shown that, no matter how the steam may be used, in the compound engine, the theoretical work is the same as it would be if the same steam were used in a simple engine, with the same measure of expansion.

It will be observed that, in the preceding table, the ratio of the resultant terminal pressure to the resultant initial pressure is

$$\frac{0.83}{2} = 0.415\,;$$

while in the case of the simple engine this ratio is only

$$\frac{0.5}{3.0} = 0.1667.$$

73. *Work of the Steam in the Compound Engine Determined Analytically.*—

(*a*). When the steam is used without expansion in the *h. p. cyl.*

In Fig. 25, let

unity = area of the *h. p. p.*
A = " " " *l. p. p.*
P = total steam pressure on the steam side of the *h. p. p.*

$w =$ the work of the steam on the steam side of the *h. p. p.* during a stroke from right to left.

$w_1 =$ the work of resistance on the exhaust, or left, side of the *h. p. p.* during the same stroke.

$w_2 =$ the work on the steam side of the *l. p. p.* during the same stroke.

Then, it will be observed, the measure of expansion will be A.

We shall have, for the resultant work of the steam, per stroke,

$$W_r = w - w_1 + w_2 \dots\dots\dots\dots\dots\dots (a)$$

Now the works w_1 and w_2 are works performed during expansion; and since, in such cases, that portion of the stroke done before expansion is due to the movement of the piston through a space equal to unity, the remainder of the stroke, or the space moved over by the piston during expansion, must be represented by $l - 1$.

But l is numerically equal to the measure of expansion A.

We have, therefore, for the actual stroke of our pistons, in this case,

$$A - 1.$$

The dotted rectangles on the right of each cylinder have a length, in the direction of the stroke, of unity; and heights representing the areas of, or pressures upon, the two pistons, respectively.

The lengths of the cylinders, increased by the lengths of the dotted rectangles, represent imaginary strokes, which are numerically equal to the measure of expansion; but the pistons actually move over a space equal to these distances, less one, in each case.

Now the work on the steam side of the *h. p. p.* is measured by product of the pressure P, by the actual stroke $A - 1$.

$$\therefore w = P (A - 1).$$

Again,

$$w_1 = P \log_e . A;$$

and

$$w_2 = P A \log_e . A.$$

These will be understood when attention is called to the fact that the initial pressures on both sides of the *h. p. p.* are the same

and in each case equal to P; while the initial pressure on the steam side of the *l. p. p.* is $P\,A$—since it is A times as large as the *h. p. p.*

We therefore have the following expression for the resultant work per stroke—obtained by the substitution of the values of w, w_1 and w_2 in (a).

$$\begin{aligned} W_r &= P\,(A-1) + P\,A\,log_e.\,A - P\,log_e\,A. \\ &= P\,A - P + (P\,A - P)\,log_e.\,A. \\ &= (P\,A - P)\,(1 + log_e.\,A. \quad \ldots\ldots\ldots\ldots\ldots (b) \end{aligned}$$

Now, this work is expressed in terms of a unit of stroke equal to a; and the unit of work is measured by the initial pressure, in each case, multiplied by this unit of stroke.

Hence, in order to compare this work with the work which would be done by the same steam, used with the same measure of expansion, in the *l. p. cyl.*, we must first determine the value of the unit of stroke (a') in that case, as compared with a.

The actual stroke in the second case will be A units; while in the first case it was $A - 1$ units. Hence

$$(A-1)\,a = Aa';$$

$$\therefore a' = \frac{(A-1)\,a}{A}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (c)$$

Now, this second unit of stroke being smaller than the first unit, it follows that the work represented by (b), and involving the larger unit of stroke, will contain the smaller unit a greater number of times in the ratio

$$\frac{a}{a'} = \frac{A}{A-1}$$

Multiplying, now, the coefficient of $1 + log_e.\,A$, in (b), by $\frac{A}{A-1}$ we get

$$P\,A - P \times \frac{A}{A-1} = \frac{P\,A^2 - P\,A}{A-1} = P\,A;$$

and substituting this result in the place of $P\,A - P$ in (b) we obtain, finally,

$$W_r = P\,A\,(1 + log_e.\,A); \quad \ldots\ldots\ldots\ldots\ldots (51)$$

which is, (35^a), the expression for the work per stroke of the

same steam, in a simple engine the area of whose piston is A, with the measure of expansion A.

(*b.*) When the steam is expanded in the *h. p. cyl.* Fig. 26.

The explanation already given, of Fig. 25, will apply to Fig. 26; the only difference between the two diagrams being the expansion curve in the *h. p. cyl.* of Fig. 26.

In addition to the notation employed in case (*a*), let

n = the measure of expansion in the *h. p. cyl.*
= also the number of units, in the stroke of the *h. p. p.*, used in expressing the work done on the steam side of the *h. p. p.*
In space, n units $= A - 1$ units; — in the sense that six *feet* = two *yards*.

Assume, as before, that the clearance = zero, and that there is a perfect vacuum on the exhaust side of the *l. p. p.*

The total measure of expansion will be

$$n\,A.$$

As before, we shall have, (*a*),

$$W_1 = w - w_1 + w_2. \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Now,

$$w = P\,(1 + log_e\, n); \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

in which the stroke is n units, each equal to the space traversed by the piston before expansion begins.

These n units, representing the actual stroke of the piston, are equivalent to $A - 1$ of the units of stroke which will be involved in the expressions for w_1 and w_2. We must therefore multiply (*b*) by

$$\frac{A - 1}{n}.$$

We thus get

$$w = P\,\frac{A - 1}{n}.\,(1 + log_e\, n)$$

$$= \frac{P\,A}{n} - \frac{P}{n}\,(1 + log_e\, n) \quad \ldots\ldots\ldots\ldots (c)$$

Again, since the initial resistance to the motion of the *h. p. p.* is $\frac{1}{n}$ th of P, we have

$$w_1 = \frac{P}{n} . \log_e . A.$$

$$\therefore w - w_1 = \frac{P\,A}{n} (1 + \log_e . n) - \frac{P}{n}(1 + \log_e . n) - \frac{P}{n} . \log_e . A$$

$$= \frac{P\,A}{n}(1 + \log_e . n) - \frac{P}{n} - \frac{P}{n}\log_e . n - \frac{P}{n} \log_e . A$$

$$= \frac{P\,A}{n} (1 + \log_e . n) - \frac{P}{n} - \frac{P}{n} (\log_e . A + \log_e . n)$$

$$= \frac{P\,A}{n} (1 + \log_e . n) - \frac{P}{n} (1 + \log_e . n\,A). \quad \ldots\ldots\ldots(d)$$

As before,

$$w_2 = \frac{P}{n} . A \log_e . A. \quad \ldots\ldots\ldots\ldots\ldots\ldots(e)$$

Adding (e) and (d), we get

$$W_r = w - w_1 + w_2$$

$$= \frac{P\,A}{n} . \log_e . A + \frac{P\,A}{n} (1 + \log_e . n) - \frac{P}{n} (1 + \log_e . n\,A)$$

$$= \frac{P\,A}{n} (1 + \log_e . A + \log_e . n) - \frac{P}{n} (1 + \log_e . n\,A)$$

$$= \frac{P\,A}{n} (1 + \log_e . n\,A) - \frac{P}{n} (1 + \log_e . n\,A)$$

$$= \frac{P}{n} . (A - 1) [1 + \log_e . n\,A]. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(f)$$

Now, the work expressed in (f) involves the unit of stroke which is equal to the actual stroke of the pistons, divided by $A - 1$. In other words, the stroke contains $A - 1$ units, each equal to a.

Now, in order to compare this work with the work which would be performed by the same steam, used in the *l. p. cyl.*, with a measure of expansion $= n\,A$, we must multiply the value of W_r, in (f), by the inverse ratio of the units of stroke, in the two cases, respectively.

If the unit of stroke in the last case be a', we shall have,

$$(A-1)\,a = n\,A\,a';$$

whence

$$\frac{a}{a'} = \frac{n\,A}{A-1};$$

Multiplying (f) by this ratio, we get, finally,

$$W_r = \frac{A\,n}{A-1} \cdot \frac{P\,(A-1)}{n}\,[1 + log_e.\ n\,A].$$

$$= P\,A\,(1 + log_e.\ n\,A). \quad \ldots\ldots\ldots\ldots\ldots\ldots (52)$$

Here, again, we have a result which is identical with that which would have been obtained for the work of the same steam, in a simple engine, with the same measure of expansion; or in a cylinder the area of whose piston is A, and in which the measure of expansion is nA.

Eq. (52) is general; if we make $n = 1$, it reduces to

$$W_r = PA\,(1 + log_e.A)$$

which is identical with (51).

74. *Results of Tests of Engines.*—The results which follow, have been obtained, within a few years, from careful tests of both compound and simple engines.

They are presented in this connection, not only as affording the means of comparison of the performances of different types of engines, but on account of the valuable practical information which may be derived from them.

(*a*) *Holly Quadruplex Compound Pumping Engines of the Buffalo Water-Works.*—

These results are taken from the report of a test made by R. H. Buel, C. E., in 1879.

EVAPORATION PER HORSE POWER.

Pounds of water, per total horse-power per hour,	17.53
" " " " indicated " " "	21.46
" " " " net " " "	25.51

COAL CONSUMPTION.

The coal consumption, per horse-power, was as follows:

	Coal.	Combustible.
Per total horse-power, pounds.	1.55	1.49
" indicated horse-power, pounds........	1.94	1.82
" net " "	2.31	2.17

Mean Steam Pressure.—Efficiency.

The mean pressures, in pounds per square inch, as deduced from indicator diagrams, were as follows :

Low-pressure cylinders	11 3
High " "	50.4
Pump water-pressure	47.7

The boiler pressure was 68 pounds per square inch. There were four cylinders, all of the same size, from one of which, in which the steam was expanded twice, the steam was exhausted into the other three.

The steam was therefore expanded about *six* times.

The mean indicated pressure on each steam piston was

$$\frac{11.3 \times 3 + 50.4}{4} = 21.08 \text{ pounds.}$$

The pressure on the pump-piston, reduced to the area of the steam-piston, was 17.86 pounds per square inch.

The efficiency of the mechanism was therefore

$$\frac{17.86}{21\ 08} = 0.847\,;$$

and the friction of the engine, and its load,

$$1.000 - 0.847 = 0.153.$$

The evaporation, per pound of coal, per pound of combustible, etc., was :

From temp. of feed (110°), per lb. coal	11.04
" " " " " " " comb	11.78
" " " " per sq. ft. of heating surface	1.54
" and at 212°, per lb. of coal	13.63
" " " " " " combustible	14.54
Steam pressure, above the atmosphere	68.00
Total heat-units per lb. of coal, 966 × 13.63 =	13167.
" " " " " combustible, 966 × 14.54 =	14046.

This last result indicates a remarkable boiler efficiency, and suggests the possibility of an error, either in the coal consumed, or in the determination of the total evaporation.

The duty during Mr. Buel's test was 86,176,315 foot pounds, per 100 pounds of coal ; with two boilers in use and with a rate of combustion equal to 6.45 pounds of coal per square foot of grate, per hour.

Some months earlier, the writer made a duty test of the same engine, using one boiler, with a rate of combustion equal to 14.5 pounds, and obtained a duty of 80,000,000 foot-pounds per 100 pounds of coal,

(*b.*) *Perkins' Machinery of the Steamer Anthracite—Three-Cylinder Compound Engine.*—

This engine was constructed for the purpose of testing the efficiency of extremely high pressures and large measures of expansion. During the trial, steam was carried at 225 pounds, and was expanded about 25 times.

The trial was had Aug. 13th and 14th, 1880, and with the following results:

Pounds of water per horse-power per hour,

Per total horse-power	18.28
" indicated "	21.68
" net "	24.62

Evaporation per pound of coal, and per pound of combustible.

Per pound of coal, from 100°	8.3	lbs.
" " " combustible, from 100°	10.07	"
" " " coal from and at 212°.	9.27	"
" " " combustible, from and at 212°.	11.25	"

Steam pressures, cut-off, and expansion.

Steam pressure in boiler, above atmosphere.	216.5	lbs.
" " initial 1st cylinder	201.64	"
Cut-off in 1st cylinder	0.52	
Initial pressure in 2d cylinder	60.	lbs.
Cut-off in 2d cylinder, about	0.50	
Initial pressure in 3d cylinder	27.71	lbs.
Cut-off in 3d cylinder	0.28	
Terminal pressure in 3d cylinder	9.55	lbs.
Steam expanded, times	25.7	

From the foregoing we may deduce the following:

Pounds of coal per total horse-power per hour,

$$= \frac{18.28}{8.3} = \ldots\ldots\ldots\ldots 2.20$$

Pounds of coal per indicated horse-power per hour,

$$= \frac{21.68}{8.3} = \ldots\ldots\ldots\ldots 2.61$$

Pounds of coal per net horse-power per hour,

$$= \frac{24.62}{8.3} = 2.97.$$

In Art. 49, it was shown that, with steam of one atmosphere, and without expansion, the quantity of water or steam necessary to maintain a total horse-power, an hour, was 35.7 pounds.

In this example, with an enormous steam pressure, and with an expansion of 25.7 times, in three cylinders, we find the quantity of steam necessary to maintain a total horse-power, an hour, to be 18.28 pounds; or about one-half the theoretical cost of a total horse-power, with low steam and without expansion.

The saving, then, in this extreme modern case, is only about 50 per cent.

(*c*). *Holly Quadruplex, Non-Compound, Pumping Engines of the Troy Water-Works.*—

The trial of these engines was had, Aug. 3d and 4th, 1880. The water supplied to the boilers was not weighed; and for this reason we cannot give all of the results which it would be interesting to know. The following facts, however, will enable us to form a fair judgment as to the relative merits of this particular type of engine, when compounded, and when used as a simple engine:

Steam pressure, above atmosphere........	75. lbs.
Cut-off, about...........................	0.25
Coal, per sq. foot of grate, per hour.......	7.73 lbs.
Piston speed, feet per minute.......	90.
Duty, per 100 lbs. coal.....................	84,960,000.

(*d*). *E. P. Allis & Co.'s Three Cylinder Compound Pumping Engines of the Allegheny City, Pa., Water-Works.*—

The following are the results of a test of engine No. 2, by the writer, Sept. 1st and 2d, 1884:

Duration of test, hours.........................	24.
Pounds of bituminous coal burned...............	18020.
" " water evaporated.....................	120995.
" " " " per pound of coal.....	6.714
" " coal burned per square foot of grate, per hour..........................	11.376
Mean steam pressure, above atmosphere..........	109.29
" pressure in receiver.......................	23.36
" vacuum, inches, about.....................	25.
" temperature of feed-water.................	81.°66

Steam expanded, times.	19.555
Mean net horse-power of engine	259.423
" pounds of coal per net horse-power, per hour,	2.893
" " " steam " " " " "	19.432
" duty, in foot-pounds, per 100 pounds of coal,	68,227,093.
" duty, based upon an evaporation of 10 pounds of water per pound of coal, according to the terms of the contract	106,368,339.
Mean pounds of coal, per net horse-power per hour, estimated on the same basis....	1.964

Collecting results, we have, for the number of pounds of steam required to maintain a net horse-power, in each of five cases, the following:

1. Holly quadruplex, Buffalo...................... 25.51
2. Perkins, three-cylinder compound............... 24.62
3. Allis & Co., " " No. 1.......... 18.568
4. " " " " " No. 2.......... 19.432
5. Gaskill, compound, Saratoga, N. Y.............. 19.015

The measures of expansion, in each of these five cases, were as follows:

In No. 1, about	8.	times.
" " 2,	25.7	"
" " 3,	17.839	"
" " 4,	19.555	"
" " 5,	13.33	"

The full results of the tests of Nos. 3 and 5 have not been included with the others.

The theoretical quantity of water required, per net horse-power per hour, under the conditions of No. 3, has been calculated by (50), Art. 69, and found to be 11.967 pounds. But it was also found that only 66.02 per cent. of the water actually evaporated was accounted for at the exhaust of the low-pressure cylinder; the other 33.98 per cent. having been condensed in the cylinders.

Taking the steam which remained, at the instant of final exhaust, as representing the quantity theoretically necessary, as

indicated by Eq. (50), we have for the total steam, including that which was condensed,

$$\frac{11.967}{0.66} = 18.13 \text{ lbs.};$$

while the actual quantity was 18.568 pounds.

The loss due to condensation in the cylinder, which increases with the measure of expansion, is a serious one, which renders necessary an uncertain modification of any possible theoretical conclusion. It can only be determined by actual test, in each particular case.

The method of determining the cylinder condensation will be explained and illustrated in another place.

VIII. The Indicator.

75. *Description.*—The steam engine indicator, Fig. 26½, is a small, single-acting engine, which is attached, alternately, to both ends of the steam cylinder, for the purpose of indicating the extent and character of the work of the steam, during an outward and inward stroke of the piston.

It consists of a cylinder, having a sectional area of half a squâre inch, in which moves a piston having a stroke which varies with the pressure of the steam. On top of the piston, and between it and the cylinder head, is placed a spiral spring, inclosing the piston rod, which opposes and measures the pressure of the steam on the under side of the piston. The piston rod of the indicator extends upward, through the cover of the cylinder, and at its upper extremity connects with a system of levers which carry a sharp-pointed pencil. Upon an arm attached to the indicator cylinder, and with its axis parallel to that of the latter, is placed a cylinder, about two inches in diameter, and about four inches long, around which is wrapped a piece of paper, and which revolves about its axis, against the tension of a spiral spring, somewhat less than a complete revolution.

A cord, wrapped about a grooved pulley at the bottom of this cylinder, leads to and is connected with an arm of a vibrating lever, or to an oscillating arc, which has a motion coincident with the piston of the main engine.

Each instrument is provided with several springs; of different capacities, which are stamped with figures, 10, 20, 30, 40, 60 and 80, indicating the capacity of the individual spring.

Thus, a spring having 40 stamped upon it is designed to be used in connection with pressures up to about 80 lbs. per square inch; and when it is used, the pencil rises at the rate of an inch for 40 pounds Similarly, the ultimate capacity of each spring is about twice the pressure indicated by the number stamped upon it; while the scale of the movement of the pencil is indicated by the number.

Referring, now, to Fig. 26ª, which shows the indicator, attached to pipes connecting the ends of the main cylinder, and the cord, *g h*, leading from the grooved pulley *g*, to the oscillating arc, *a b*. *e* is a stud on the cross-head, and *e d* is a link-rod, connecting the stud *e* with the end *d* of the lever *d h*, which oscillates about *p* as a centre.

76. *Operation of the Indicator.*—Now, suppose the piston *P*, of the engine, to be at the left-hand end of the cylinder, and let steam be admitted. The piston will then begin to move toward the right, and with it the cross-head and stud *e*, and the end *d* of the oscillating arm.

As the arc *a b* revolves to the left, the elasticity of the spring, coiled within the cylinder *I*, will keep the cord *h g* taut, and will cause the cylinder to revolve from right to left, carrying with it the paper which is wrapped upon its surface, and upon which the point of the pencil is resting.

At the instant that steam is admitted—*k'* being open, and *k* closed—the steam, acting on the under side of the indicator piston, raises it, and with it the pencil, to a height which measures the pressure of the steam above the atmosphere, and indicates it by a vertical line on the paper.

As the piston moves, and as the cylinder *I* revolves under the point of the pencil, a horizontal line is drawn on the paper. This horizontal line will be continued as long as the motion continues; or until the piston reaches the end of its stroke. Then, the exhaust being opened, the pressure will instantly fall to the atmosphere, and with it the piston of the indicator, and the pencil, which will make another vertical mark, downward, on the paper.

Steam is now admitted to the right-hand end of the cylinder, which, since *k* is closed, has no communication with the indicator; the piston, stud *e* and arm *p d*, move toward the left, and the arc *a b* revolves toward the right, carrying the cord *h g* with it, and causing the cylinder *I* to make a part of a revolution, from

left to right, and causing the pencil to trace a horizontal line from right to left, on the paper, and terminating at the starting point. Fig. 27.

The pencil of the indicator has thus traced a rectangle, on the paper, which is designated an "indicator card."

This rectangular card shows that the steam followed full stroke; its height indicates the steam pressure per square inch on the piston, and its length indicates the stroke of the piston.

The *area* of the diagram, which is measured by the product of its altitude by its base, or by the product of the pressure per square inch by the stroke, therefore represents the work done on each square inch of the piston, during the stroke from left to right.

A corresponding card, or diagram, would be obtained from the other end of the cylinder, by closing k' and opening k.

77. *Card from a Condensing Engine.*—In the case of a condensing engine, where the steam is exhausted into a condenser, in which a partial vacuum is maintained, the piston of the indicator will be forced below its normal position, by the atmospheric pressure above it—or by the excess of the atmospheric pressure over the pressure of the uncondensed vapor in the condenser—and the pencil will, therefore, during the return stroke, trace a line *below* the lower line of the diagram described in Art. 76. See, also, Fig. 28.

If both k and k' be closed, and the pencil be made to trace a line on the paper, the line so traced is called the "atmospheric line."

A line drawn parallel to the atmospheric line, and at a distance below it equal to the pressure of the atmosphere, in pounds per square inch, according to the scale of the indicator spring, is called the "vacuum line." Fig. 28.

78. *Card from a Condensing Engine in which Steam is Used Expansively.*—

If the admission of steam to the cylinder be suppressed when the piston has completed a portion of the stroke, and the inclosed steam be then allowed to expand, the pressure will fall, as the piston continues its stroke, and the indicator piston with the pencil will fall with it, the latter describing the curved "expansion line" which was shown in Fig. 20, Art. 66.

The steam being exhausted, at the end of the stroke, the

pencil will descend, describing a vertical line, which will extend below the atmospheric line, to a distance depending upon the vacuum in the condenser.

During the return stroke, the indicator piston being held down by the excess of the atmospheric pressure over the tension of the vapor in the condenser, the pencil will trace a horizontal line, which will terminate at the place of beginning, at the end of the return stroke, and which will complete the diagram.

This diagram is represented in Fig. 29, in which the trace of the diagram begins and ends at the steam corner A; $A\,B$ is the admission line; $B\,C$ is the steam line; $C\,D$ is the expansion line; D is the exhaust corner; $D\,E$ is the exhaust line, and $E\,A$ is the vacuum line.

$a\,a$ is the atmospheric line, and $v\,v$ the perfect vacuum line.

79. *The Indicator Motion.*—Indicator motions are secured in a variety of ways, depending upon the conditions and surroundings in each particular case.

In general, the object to be attained is a motion of the card on the oscillating cylinder I, Fig. 26, which will be exactly coincident with the stroke of the steam piston of the engine.

A convenient motion for stationary or other horizontal engines is shown in Fig. 26. The motion of the piston, which is the same as that of the cross-head and its stud e, is fixed. The length of the diagram on the card, which limits the oscillation of the cylinder I, of the indicator, is limited to about four or four and one-fourth inches; say, four and one-fourth inches.

The lengths of the arms $p\,d$ and $p\,h$, must be made proportional to the stroke of the piston s, in inches, and the length l, of the diagram; or,

$$\frac{p\,h}{p\,d} = \frac{l}{s} = \frac{4.25}{s}.$$

The centre of the oscillating arc $a\,b$, must coincide with the centre p, about which the arm $d\,h$ oscillates. The motion of the cord $h\,g$, will then be coincident with that of the steam piston—except in so far as it is varied by the changes, in direction, of the link $e\,d$.

Another method is to hang a rod from a centre attached to the ceiling over the engine, and vertically over the position of the stud e, when the piston is at mid-stroke; the rod having a slot at its lower extremity which fits a pin placed horizontally in the stud e. Attached to the rod, at a distance from its upper end,

or centre of motion, which bears the same relation to the distance from the centre of motion to the pin on the stud *e*, that the length of the desired diagram bears to the stroke of the piston of the engine, should be secured an arc, similar to *a b*, Fig. 26, having its centre at the centre of motion of the rod. From this arc, a cord should lead directly to the indicator guide pulley.

The vibrations of the rod will then communicate, through the cord, the proper motion to the cylinder *I* of the indicator.

Still another method consists in attaching a cord to an arm on the cross-head, leading it thence over a pulley, on one of the cross-head guides, and thence to the larger of a pair of differential pulleys, supported on the floor of the engine-room. From the smaller of the two differential pulleys, another cord is led directly to the indicator. The diameters of the larger and smaller differential pulleys should be proportional to the stroke of the engine piston and to the length of the indicator diagram, respectively.

80. *Modifications of the Form of the Indicator Diagram Due to Practical Conditions.*—

The diagrams heretofore referred to are simply ideal diagrams. In Fig. 29, for example, the diagram indicates that steam is admitted at the instant of the completion of a return stroke, and that the full steam pressure, represented by the steam line *A B*, is obtained instantly; that the steam pressure continues, absolutely uniform to the point of cut-off *c*, and that the closure of the steam valve and cut-off are effected instantaneously; that the expansion continues to the entire completion of the stroke, and that the exhaust occurs instantaneously, at the end of the stroke, and before the beginning of the return stroke; and, finally, that the exhaust, or vacuum line is a straight line, parallel to the atmospheric line.

None of these conditions can be satisfied, nor can any of these results be attained in practice.

Fig. 30 shows a diagram as modified by practical conditions. The steam corner *A* is rounded because of a slight compression in the exhaust, due to the closing of the exhaust before the completion of the preceding return stroke. As soon as the exhaust is closed, a small volume of vapor is confined in the cylinder, on the exhaust side of the piston, which is gradually compressed into a smaller volume, during the completion of the stroke, and thus has its tension increased.

The corner B is also slightly rounded, because of the motion of the piston before the indicator piston feels the full effect of the steam.

When the steam or cut-off valve is closed instantaneously, the angle C is sharp and well defined; but when the steam port is closed gradually, as it is when the slide valve is used, the pressure begins to fall, as the rate of admission of the steam is reduced, and continues to fall—the pencil tracing the line *si*—, until the steam port is wholly closed. Up to this point the curve *si* is *convex* upward; but beyond this point, and while the pressure is falling, in consequence of the expansion of the steam, the curve becomes *concave* upward.

The change in curvature, therefore, marks the point at which the steam port is wholly closed, and the admission of steam wholly suppressed.

The corner D is rounded, because the exhaust port must begin to open—permitting the escape of steam from the cylinder—, before the completion of the stroke.

The corner E is rounded, because the steam cannot wholly escape until the return stroke has commenced, and has been partly completed.

81. *Interpretation of Indicator Diagrams.*—The extent and character of the modifications of the form of the indicator diagram, by the greater or less rounding of the several corners of the theoretically perfect diagram, Fig. 29, tell the whole story as to the adjustment and operation of the valve and valve-motion, and of the behavior of the steam in the cylinder.

To illustrate our meaning more clearly, reference is made to the following diagrams, with their explanations:

In Fig. 31, the position and character of the line *ss* indicates that the steam-valve opened too early.

In Fig. 32, the position and character of the line *ss* indicates that the steam valve opened its port too late.

In Fig. 33, the line *ee* indicates that the exhaust port was opened too early.

In Fig. 34, the line *ee* indicates that the exhaust port was opened too late.

In Fig. 35, *vv* indicates the use of a slide-valve; the cut-off being effected by "lap" on the steam side, and excessive cushioning—indicated by the line *cc*—, due to the early closing of the exhaust port.

In Fig. 36, the line *t t* indicates that the steam supply was regulated by the throttle, and that the engine was running rapidly; or, that the spring of the indicator was weak.

In Fig. 37, we have a diagram from a non-condensing engine, in which the expansion was carried below the atmospheric pressure.

In Fig. 38, we have a fair diagram from a condensing engine, with a slide valve, and with a cut-off effected by "lap" on the steam side.

Fig. 39 indicates that the diagram was taken from a non-condensing engine, in which there was excessive back-pressure, due, probably, to a small exhaust port; or, possibly, to the exhaust being through a feed-water heater.

Fig. 40 represents a diagram from a condensing engine, with a nearly instantaneous cut-off and a leaky piston; the latter condition being indicated by the fact that the pressure, during expansion, fell largely below the theoretical pressure, giving the theoretical expansion line *e e*.

In Fig. 41, the fact that the expansion line lies considerably above the theoretical expansion curve, indicates a leaky steam valve, which permitted the entrance of steam to the cylinder, after it was closed.

A very great variety exists, in the diagrams of different engines, under their various conditions, and many more examples might be given; but it is believed that enough have already been given to illustrate, in a general way, the principles governing their interpretation.

81½. *The Total, Indicated and Net Works.*—In Fig. 42, *a a* being the atmospheric line, and *v v* the perfect vacuum line, let us consider the pressures, per square inch on the steam piston, at the points p and p' of the stroke of the piston.

At the point p, the total pressure per square inch, on the steam side of the piston, is represented by $p\,s$, according to the scale of the spring or diagram.

At the point p', the total pressure is represented by $p'\,s'$.

If now the length $A_1\,E_1$, of the diagram, be divided into any number of equal parts (say 20), and if lines corresponding to $p\,s$, $p'\,s'$, etc., be measured at the middle points of the n equal spaces thus formed, the sum of the lengths of the lines thus measured, divided by their number, will represent the *mean total pressure* per square inch, for the entire stroke of the piston.

Let $p_1, p_2, \ldots p_n$ represent the pressures measured. Then will the *mean* total pressure be

$$p_{m.t.} = \frac{\Sigma(p)}{n}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Or, if the area of the diagram A_1BCDE_1 be measured by means of a planimeter and the length of the diagram, A_1E_1, be determined, then will the mean total pressure—or the mean height of the diagram—be determined by dividing the area by the length. If the area be in square inches, and the length of the diagram be expressed in inches, the result will be the length of a line, in inches, which will represent the mean total pressure per square inch on the piston. If, for example, the result be 1.375 inches, and the scale of the diagram be 40, then

$$p_{m.t.} = 40 \times 1.375 = 55 \text{ pounds.}$$

Again, while p, the mean total pressure, is represented by ps, the back pressure, on the opposite or exhaust side of the piston, will be represented by $p\,b$, and the effective pressure *on the piston*, or the *indicated*, pressure, will be

$$p_i = bs = ps - pb. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

If, then, bs, $b's'$, etc., be measured at the middle of each of the n spaces on the diagram, we shall have the *mean indicated* pressure, per square inch on the piston,

$$p_{m.i.} = \frac{\Sigma(p_i)}{n}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

If $p_{m.i.}$ be multiplied by the length of the diagram—both being expressed in inches—the product will evidently be the area of the diagram $A\ B\ C\ D\ E$, traced by the pencil of the indicator. This area may be measured with a planimeter, as already suggested, and the mean indicated pressure determined by dividing it by the length of the diagram in inches.

The mean back pressure will be

$$p_{m.b.} = p_{m.t.} - p_{m.i.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (d)$$

If p_f represent a mean pressure per square inch on the piston, just sufficient to overcome the friction of the engine *per se*, together with the additional friction due to the load on the engine, we shall have the mean *net*, or commercially valuable, pressure,

$$\begin{aligned} p_{m.n.} &= p_{m.i.} - p_f. \\ &= p_{m.t.} - (p_{m.b.} + p_f). \quad \ldots\ldots\ldots\ldots\ldots (e) \end{aligned}$$

Similar values of the mean pressures should be found from a diagram, taken at the same time, from the other end of the cylinder.

Means of the corresponding mean pressures, during the two strokes of a revolution of the engine should then be taken.

Let

$P_{m.t.} = \dfrac{p_{m.t.} + p'_{m.t.}}{2}$

= mean total pressure per square inch on the piston during a revolution of the engine.

$P_{m.b.}$ = mean back pressure,
$P_{m.i.}$ = " indicated pressure, and
$P_{f.}$ = " friction pressure.
a = area of the piston in square inches.
s = stroke of piston, in feet.
n = number of revolutions, per minute.

We shall then have for the "total" horse-power,

$$H\text{-}P_t = \frac{P_{m.t.}\,a\,n\,2\,s}{33000}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (53)$$

for the "indicated" horse-power,

$$H\text{-}P_i = H\text{-}P_t.\frac{P_{m.i.}}{P_{m.t.}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

and for the "net" horse-power,

$$H\text{-}P_n = H\text{-}P_t.\frac{P_{m.n}}{P_{m.t.}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

(54) and (55) follow from (53), for the reason that, other things being the same, the powers will be directly proportional to the mean pressures.

The efficiency of the engine and connected machinery, will evidently be,

$$E_m = \frac{H\text{-}P_n}{H\text{-}P_t} = \frac{P_n}{P_t}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

This efficiency will, obviously, be increased by increasing P_n; or, by diminishing $P_{m.b.}$ and P_f.

82. *Distribution of Power.*—In order to illustrate the distribution of power in any case, the following example is extracted from Chief Engineer Isherwood's "Engineering Precedents:"

DISTRIBUTION OF POWER OF THE BRITISH WAR SCREW STEAMSHIP "CONFLICT."

	H-P.		Per Cent.
Gross indicated horse-power developed by the engines	912.96		
Power required to work the engine *per se*	80.61		
Power applied to the screw shaft	832.35	or	100.00
Power expended in overcoming the friction of the load	62.42		7.5
Power expended in overcoming the cohesive resistance of the water by the screw blades	26.91		3 20
Power expended in the slip of the screw	193.56		23.26
Power expended in the propulsion of the vessel	549.46		66.01
Totals	832.35		100.00

This distribution is somewhat different from that which we have indicated, but will, nevertheless, serve to illustrate the method and the meaning of a proper distribution.

It would seem that, instead of comparing the net or useful work with the indicated power, less the power required to work the engine *per se*, the comparison of the former should have been made with the *total* work or power of the steam on the steam side of the piston. The power required to overcome the back pressure and the friction of the engine *per se*, are certainly both variable quantities, which are dependent, in a greater or less degree, upon the perfection of the design and adjustment of the machinery, and should, therefore, be included as variable elements of the efficiency of the mechanism.

The power expended in the slip of the screw, constitutes an element in the distribution which is peculiar to the engines of steam vessels. Had this engine been employed in driving the machinery of a mill or manufactory, this power would have been useful power, and the efficiency of the mechanism would, in that case, have been proportionately greater, or $66.01 + 23.26 = 89.27$ per cent.

83. *Indicated Power of a Non-Condensing Engine.*—In Fig. 43, let

W_t = the total work, per stroke, which is represented by the area $A_1 B C D E_1$.

W_b = work due the back pressure, which is represented by the area $A E E_1 A_1$.

W = work represented by the area $A B C F$.

W_1 = work " " " $A F D E_1 A_1$.

w = work " " " loop $F E D$.

W_i = the indicated work to be found.

Then,

$$W_i = W_t - W_b; \quad \text{.....................} (a)$$

but

$$W_t = W + W_1,$$

and

$$W_b = W_1 + w.$$

These values, substituted in (a), give

$$\begin{aligned} W_i &= W + W_1 - W_1 - w \\ &= W - w. \quad \text{.........................} (57) \end{aligned}$$

In this case the steam is expanded below the pressure of the atmosphere, and when the exhaust is opened the pressure at once rises to, and generally a little above, the atmosphere, as the return stroke begins.

Whenever the terminal pressure is at, or above, the atmosphere, there will be no loop, w will become equal to zero, and (57) will become

$$W_i = W. \quad \text{.....................} (58)$$

84. *Theoretical Curve of Expansion.*—Fig. 44. By the theoretical curve of expansion we mean the hyperbolic curve of Art. 61, Fig. 17, due to the assumption that the volume of a given quantity of steam v, varies inversely with its tension, or pressure p; or, that the product $p v$ is constant.

In Fig. 44, $A E$ is the length of the ideal indicator diagram, and represents, also, the stroke of the piston, or length of that portion of the cylinder, which, at the end of a stroke of the piston, is filled with steam.

$A B$ is the total initial pressure of the steam, in pounds per square inch; and represents, also, the diameter of the cylinder.

The rectangle $A\,B\,F\,E$ represents a longitudinal section of the cylinder.

$A\,B\,b\,c$ is an additional area, representing the clearance space, which is also filled with steam, with a varying pressure equal to that in the cylinder at every point of the stroke.

$A\,c$ bears the same relation to $A\,E$ that the volume of the clearance space bears to the space displacement of the piston. In other words, $A\,c$ is the altitude of a cylinder, whose diameter is equal to that of the steam cylinder, and whose volume is equal to that of the clearance space.

Now, C being the cut-off, when the piston is at d, having completed the part $A\,d$ of its stroke, there will be confined in the cylinder and clearance space, a volume v, of steam proportional to $c\,d$, and having a total pressure p.

When the piston has reached f, the volume v, of steam has expanded, and if $c\,f = 2\,c\,d$, we shall have—since the volumes are proportional to $c\,d$ and $c\,f$—a volume $v' = 2\,v$, whose pressure is p'.

The product of the pressure by the volume being constant, we have

$$p\,v = p'\,2\,v$$
$$\therefore p' = \tfrac{1}{2}\,p.$$

Similarly, when the piston reaches the point h,—$c\,h$ being equal to $3\,c\,d$—we shall have the pressure

$$p'' = \tfrac{1}{3}\,p.$$

When the piston reaches j, the pressure of the expanding steam will be

$$p''' = \tfrac{1}{4}\,p:$$

and so on, for other positions of the piston, until the stroke is completed.

If, now, the pressures p', p'', etc., be found and laid off at the points f, h, j, etc., to the scale of the diagram, as ordinates, and a fair curve be drawn through their extremities, the curve so drawn will be the theoretical curve of expansion.

The curve may be constructed *graphically*, as follows:

Complete the rectangle $A\,B\,F\,E$, taking care that its height is equal to the pressure at the instant that the steam or cut-off valve is wholly closed; construct the clearance rectangle $A\,B\,b\,c$ on the steam end of the diagram; divide the length $c\,E$ of the

diagram, into equal parts, each of which shall be equal to $c\,d$—or to $\frac{1}{2}$ or $\frac{1}{3}$ of $c\,d$,—and at the points of division f, h, j, etc., draw the perpendiculars $f\,e$, $h\,g$, $j\,i$, etc.; then draw lines $e\,c$, $g\,c$, $i\,c$, etc., and project the points p, p_1, p_2, etc., horizontally upon the corresponding perpendiculars $f\,e$, $h\,g$, etc., at the points p', p_1', p_2', etc.

Finally, draw a fair curve through e and the last named points. The curve thus drawn will be the required curve.

For, the pressures $f\,p'$, $h\,p_1'$, $j\,p_2'$, etc., are, by construction, obviously, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of the initial pressure $d\,c$; which accords with the assumption that the pressures vary, inversely, as the volumes; or, as the distances $c\,d$, $c\,f$, $c\,h$, etc., which are proportional to the volumes.

85. *Diagram from a Cylinder of the Holly Quadruplex Engine, Troy Water-Works.*—Fig. 45.

This diagram is an exceptionally good one. The steam and exhaust valves open and close quickly, and the speed of piston is only about 90 feet per minute. The clearance is 0.026, and the initial steam pressure 84 pounds above zero.

The back pressure at the beginning of the stroke, is 1.5 lbs., and at half stroke, 2.0 lbs.

It should be stated, also, that the steam cylinders are steam jacketed, on both sides and ends; the jacket steam being supplied from the main boilers.

It will be an excellent exercise to construct this diagram, from data taken from the actual diagram.

For this purpose, draw the atmospheric and perfect vacuum lines, at a distance of 14.7 lbs. apart, to a scale of 40 lbs. to an inch; lay off on the vacuum line 27 spaces of one-sixth of an inch each, and at the points of division erect perpendiculars about 2½ inches long.

The 27 spaces will represent the stroke plus the clearance; of which the stroke will constitute 26.3 parts, and the clearance 0.7 part. The steam valve closed, and the expansion began, at the end of the 8th space.

Lay off, on the perpendiculars, the actual pressures contained in the following table, and through their extremities draw a fair curve. Afterward, lay off the pressures due to the hyperbolic curve, and through their extremities draw another fair curve. The comparison is to be made between the two curves thus drawn, and it will show a remarkable agreement between the two.

The theoretical curve of expansion will lie above the actual expansion curve, from the point of cut-off to about mid stroke, and below it during the remainder of the stroke.

The conditions are as follows:

Steam, 71.5 pounds above the atmosphere.
Vacuum, 28 inches, by gauge; probably 26 inches.
Revolutions, 15.25 per minute.
Clearance, 0.026.
Cut-off, 0.239.
Expansion, 3.76 times.
Diameter of cylinder, 27 inches.
Stroke of piston, 36 inches.

$$p_m = 86.2 \frac{1 + log_e. 3.76}{3.76} = 53.76 \text{ pounds.}$$

p_m, from diagram, = 52.03 pounds.

The following table contains the

DATA FOR CONSTRUCTING ACTUAL AND THEORETICAL EXPANSION CURVES OF DIAGRAM FROM HOLLY QUADRUPLEX ENGINE—SIX MILLIONS CAPACITY—TROY WATER-WORKS:

Number of Ordinate.	Total Pressure.		Number of Ordinate.	Total Pressures.	
	Actual lbs. sq. in.	Due to Hy-perb'c Curve lbs. sq. in.		Actual lbs. sq. in.	Due to Hy-perb'c Curve lbs. sq. in.
1	86.5	.....	15	42.	42.
2	"	.....	16	39.6	39.2
3	"	.....	17	37.	36.75
4	"	.....	18	35.	34.6
5	"	.....	19	33.2	32.66
6	"	.....	20	31.2	30.95
7	"	.....	21	30.	29.4
8	84.	84.	22	28.5	28.
9	72.2	73.5	23	27.1	26.72
10	64.	65.32	24	26.	25.13
11	59.	58.8	25	25.1.	24.5
12	54.	53.45	26	24.	23.52
13	49.	49.	27	23.5	22.61
14	45.3	45.2	28	22.5	21.78

As already intimated, the pressures in the second and fifth columns have been taken by measurement from a diagram actually taken from one of the cylinders of the engine, with an indicator whose spring indicated 40 lbs. to the inch.

The determination of decimals of a pound was, of course, difficult, and depended largely upon estimate.

86. *Water Accounted for by the Volume and Pressure of the Steam in the Cylinder.*—

(*a*). In the simple engine, there are two principal points or positions of the piston, which permit the determination of the quantity of water due to the steam in the cylinder. These are first, at the point of cut-off; and, second, at the point of release, or the point at which the exhaust begins to open.

The *quantity* of steam in the cylinder is the same in both cases; but the *pressures* are different. Both volumes and pressures are easily determined from the indicator diagram—the diameter of the cylinder and the clearance being known.

Having found the volumes and pressures, the volume and weight of the equivalents, in water, may be taken from the table of Art. 40.

(*b*). In the compound engine, there are *four* points at which the volumes and pressures of the same quantity of steam may be determined.

First—At the point of cut-off in the high-pressure cylinder.

Second—At the point of release, in the high-pressure cylinder.

Third—At the point of cut-off, in the low-pressure cylinder, and

Fourth—At the point of release, in the low-pressure cylinder.

The *third* point exists only where there is a receiver, into which the high-pressure cylinder exhausts, and from which the low-pressure cylinder takes its steam.

Where the steam is exhausted from the high-pressure cylinder directly into the low-pressure cylinder, there is, usually, no cut-off in the low-pressure cylinder.

Strictly, of course, the comparison may be made at any point of the stroke; and it may be desirable, in the practical study of this question, to make the comparison between the quantities of steam, and the water accounted for by the steam, at several points in each cylinder.

It is clear that the quantity of water represented by the steam in the high-pressure cylinder, at the instant of cut-off, should be accounted for at every subsequent period, and finally, at the instant of final release, or exhaust, into the condenser, or into the atmosphere.

Generally, the quantity of water accounted for by the steam will be less, at the point of cut-off, than that put into the boiler —indicating the presence of water, with the steam, in the cylinder. At the point of exhaust, the water accounted for by the steam will be greater than that accounted for at the cut-off—indicating the reëvaporation of a portion of the water during expansion.

At the beginning of the stroke, or at the point of cut-off, in the low-pressure cylinder, the water accounted for will be less than that accounted for at the point of release in the high-pressure cylinder—indicating condensation.

Finally, at the point of release, in the low-pressure cylinder, the quantity of water accounted for by the steam is greater than that accounted for at the beginning of the stroke, or at the point of cut-off in that cylinder—indicating reëvaporation during the period of expansion.

Example.—The following data and results are taken and deduced from the performance of the E. P. Allis & Co. three-cylinder compound engine, No. 1, of the Allegheny City, Pa., water-works, during a duty test, in September, 1884:

Mean quantity of water supplied to the boilers per minute, pounds	79.8621
Mean revolutions of engines, per minute	15.403
Mean initial steam pressure, total	122.68
Area of high-pressure piston, inches	733.944
Mean volume of steam admitted to the cylinder, per revolution, equals area of piston multiplied by feet	2.67

Therefore, at the point of cut-off in the high-pressure cylinder, the mean volume of steam accounted for, per minute, was

$$2.67 \times 15.403 \times \frac{733.944}{144} = 209.612 \text{ cubic feet.}$$

At the total pressure of 122.68 pounds per square inch, the volume of steam is 223.53 times that of the original water at 39.1° Fahr., at which the weight of a cubic foot of water is 62.425 pounds.

It follows that the water accounted for was

$$\frac{209.612 \times 62.425}{223.53} = 58.455 \text{ pounds.}$$

The percentage of the water which was thus accounted for, was therefore

$$\frac{100 \times 58.455}{79.862} = 73.2;$$

indicating the condensation of 26.8 per cent. of the steam—less water entrained in the steam, plus about 4 per cent. condensation in the jackets.

At the *exhaust* of the high-pressure cylinder, the mean total pressure was, pounds........................	40.5
Mean volume of steam at point of release, cubic feet, per minute....................................	662.595

The volume of steam at a total pressure of 40.5 pounds, is 620.27 times that of the original water at 39,1° Fahr.

The water accounted for, therefore, at the release, was,

$$\frac{662.595 \times 62.425}{620.27} = 66.681 \text{ pounds}$$

per minute, or

$$\frac{100 \times 66.681}{79.862} = 83. \text{ per cent.}$$

The condensation at this point was, therefore, 17 per cent., of which 4 per cent. was condensed in the jackets; leaving 13 per cent. as the cylinder condensation at this point. The reëvaporation, during the period of expansion, was $66.68 - 58.45 = 8.23$ pounds.

The condensation in the high-pressure cylinder, before the cut-off, appears to have been $26.8 - 4 = 22.8$ per cent., or 18.2 lbs.

The reëvaporation of 8.23 lbs., in the cylinder, during expansion, leads us to conclude that, at the end of the stroke, there remained $18.2 - 8.23 = 9.97$ lbs. of water in the cylinder, which went to the receiver with the exhaust steam.

At the point of cut-off in the low-pressure cylinders (2), the quantity of water accounted for, is determined as follows:

Aggregate area of low-pressure pistons, square feet......	19 88
Mean distance through which steam follows, plus clearance, per revolution, feet..........................	1.466

The mean volume of the steam per minute, was, therefore,

$$19.88 \times 15.403 \times 1.466 = 449.029 \text{ cubic feet.}$$

The mean total pressure of the steam, at this point, was 42.45 lbs., at which the specific volume is 594.31.

The water accounted for was, therefore,

$$\frac{449.029 \times 62.425}{594.31} = 47.168 \text{ pounds}$$

per minute; which is

$$\frac{100 \times 47.168}{79.862} = 59.06 \text{ per cent.}$$

of the water supplied to the boilers.

It follows, then, that $66.68 - 47.17 = 19.51$ pounds, *certainly*, and $66.68 + 9.97 - 47.17 = 29.58$ pounds, *probably*, went to the receiver, and thence to the low-pressure cylinders, each minute —less so much as was trapped off, from the receiver.

At the release in the low-pressure cylinders, the mean volume of steam, per minute, was 2449.693 cubic feet, at a mean total pressure of 7.85 pounds. At this pressure, the specific volume of steam is 2900.

The water accounted for, per minute, was, therefore,

$$\frac{2449.693 \times 62.425}{2900} = 52.73 \text{ pounds;}$$

or,

$$\frac{100 \times 52.73}{79.862} = 66.02 \text{ per cent.}$$

of the water supplied to the boilers, was accounted for, at the final release of the steam from the low-pressure cylinders.

The reëvaporation in the low-pressure cylinders appears to have been $52.73 - 47.17 = 5.56$ pounds per minute; leaving $79.86 - 52.73 = 27.13$ pounds per minute, or 33.98 per cent. of the water to be trapped from the jackets of the three cylinders and from the receiver, and to pass with the steam from the low-pressure cylinders to the condenser, with the final exhaust.

Possibly 5 per cent. of the entire water, or a little more than one-seventh of that which disappeared between the boiler and the final exhaust, was entrained in the steam, leaving about 29 per cent. as the aggregate of condensation in the jackets and

in the receiver, together with that which finally passed to the condenser with the steam.

It is to be regretted that circumstances did not permit the determination of the quantities of water trapped from the jackets and the receiver.

In the following table, are presented the foregoing, and similar determinations for other engines:

No. 1, is the Allegheny engine, just considered, which, as already stated, is a three-cylinder compound, with receiver.

No. 2, is a Gaskill compound engine, at Saratoga, N. Y., without receiver.

No. 3, is a simple condensing pumping engine, at Decatur, Ills.

No. 4, is a simple condensing engine, No. 5, is a Harris-Corliss, No. 6, a Reynolds-Corliss, and No. 7, a Wheelock engine—all simple condensing engines—, and Nos. 8, 9 and 10, are the same engines, run non-condensing, which were tested at the Cincinnati Millers' Exposition in June, 1880, by Mr. J. W. Hill.

WATER ACCOUNTED FOR IN THE CYLINDER.

No.	Steam pressure above zero	Measure of Expansion.			Lbs. water supplied to boilers per min.	Per Cent. of Water Accounted For.			
						In *H. P. Cyl.*		In *L. P. Cyl.*	
		h. p. cyl.	*l. p. cyl.*	Total		at cut-off	at exh'st	at cut-off	at exh'st
1.	122.68	3.14	5.73	17.95	79.862	73.2	83.0	59.06	66.02
2.	83.98	3.364	3.96	13.33	27.083	63.4	80.3	66.63*	88.6
3.	81.29	6.978	....	6.978	30.383	55.16	80.	.....	.....
4.	77.29	7.9	....	7.9	......	71.9	89.96	.....	.....
5.	110.81	6.95	....	6.95	53.44†	.....	71.03	.....	.....
6.	110.55	6.67	....	6.67	56.01†	.....	72.20	.....	.....
7.	110.97	6.35	....	6.35	51.41†	.....	71.45	.....	.....
8.	104.24	6 35	....	6.35	53.51†	.....	75.35	.....	.....
9.	104.71	5.42	....	5.42	53.32†	.....	72.83	.....	.....
10.	104.12	5.14	....	5.14	56.15†	.....	75.73	.....	.....

*At beginning of low-pressure stroke.
†Steam supplied to the cylinders per minute.

The results contained in this table show, very clearly, that the power, due, theoretically, to a given weight of steam, whether used with or without expansion, cannot, by any means, be realized in practice.

These results show, too, that the gains, theoretically due to expansion, cannot be realized in practice, and that Eq. (50) must give results essentially too small.

In a general way these, and other similar, results show the

extent to which theoretical results must be modified under practical conditions.

Each engine, and each set of conditions, will require a coefficient of its own, the proper value of which can only be determined by the exercise of sound judgment, based upon experience, as well as upon study, and a careful and intelligent consideration of the practical conditions under which steam is to be employed.

Below are given the number of pounds of steam supplied to each of the three engines, Nos. 5, 6 and 7, per indicated horse-power, per hour, at the Cincinnati Millers' Exposition; also the number of pounds accounted for by the diagrams in each case:

Name of Engine.	Lbs. of Steam per Indicated Horse-power, per hour.			
	Condensing.		Non-Condensing.	
	Steam Actually Supplied.	Steam Accounted for by Diagrams	Steam Actually Supplied.	Steam Accounted for by Diagrams
Harris-Corliss	19.364	13.755	22.054	18.013
Reynolds-Corliss ..	19.489	14.886	23.933	18.896
Wheelock	19.265	13.915	24.926	19.674

Referring to the measures of expansion, in the preceding table, and to the number of pounds of steam per indicated horse-power per hour, when the engines were operated as condensing engines, as well as to the pressures, compare the results with those given by Eq. (50), Art. 69, for the theoretical costs of a *net* horse-power, in pounds of steam, per hour. See, also, Art. 74.

In the Cincinnati experiments, when the engines were operated as condensing engines, the quantities of steam per *net* horse-power, were 14.1, 13.8 and 10 per cent., greater, respectively, than the cost per *indicated* horse-power. The probable costs, per *net* horse-power, as indicated by the diagrams, were, therefore, as follows:

Harris-Corliss,	$13.755 \times 1.141 = 15.694$	pounds.
Reynolds-Corliss,	$14.886 \times 1.138 = 16.940$	pounds.
Wheelock,	$13.915 \times 1.100 = 15.306$	pounds.

87. *Compound Engine Diagrams.*—Figs 22 and 24, Art. 71, show the *combined* effect of the steam in both cylinders. They do not

show the forms or proportions of the actual diagram taken from either cylinder.

In illustrating the diagrams furnished by the high and low-pressure cylinders, respectively, it will be assumed, first, that the exhaust from the high-pressure cylinder is led directly into the low-pressure cylinder; the diameter of the latter being twice that of the former. It will also be assumed that the steam is expanded about three times, in the high-pressure cylinder.

Fig. 46 represents the diagram from the high-pressure cylinder. It is in all respects like the diagram of the simple engine, except that the curve $E\ A$, instead of indicating the atmospheric or vacuum line, indicates the curve of varying pressure on the exhaust side of the high-pressure piston, or in the low-pressure cylinder.

Fig. 47 represents the diagram furnished by the low-pressure cylinder, which, it will be observed, is identical with that part of Fig. 46, which lies below the curve $E\ A$,—the same indicator spring being used in both cases.

Fig. 48 shows the two diagrams combined, by construction, so as to indicate the form and characteristics of an ideal diagram from a cylinder having a diameter equal to that of the high-pressure cylinder, and with a piston stroke such that the space displacement of the piston will be equal to the displacement of the low-pressure piston, increased by the volume of clearance space at one end of the low pressure cylinder;—the clearance in the hypothetical cylinder being the same as that of the high-pressure cylinder, the same steam being used, and with the measure of expansion which results from the use of the two cylinders combined, as a compound engine.

Upon this diagram, may be constructed the actual expansion line, the theoretical expansion line, and, if desired, the *adiabatic* curve of expansion.

The area $A\ B\ C\ D\ E$ represents the indicated work of the steam per stroke, per square inch, on the high-pressure piston, and $A\ E\ E_1\ A_1$ the work of the back-pressure, per square inch during a stroke of the piston. The sum of these areas, or $A_1\ B\ C\ D\ E_1$, represents the total work of the steam in the high-pressure cylinder, per square inch, per stroke, which is equal to the total work per square inch, on our hypothetical cylinder,

during the portion of its stroke which is equal to the common stroke of the pistons of the compound engine.

The area on the right represents the total and indicated works of the steam on each square inch of the hypothetical engine during the remainder of its stroke, which is equivalent to the works of the steam on *four* square inches of the low-pressure piston of the compound engine. This follows, because, while, the mean pressures per square inch are the same, in both cases, that part of the stroke of the piston in the hypothetical engine during which this work is performed, is four times that of the real engine.

The work represented by space c', between the two parts of the diagram, is that due to the clearance in the low-pressure cylinder; which, inasmuch as it is not performed, is partly a loss.

To construct this diagram, draw an indefinite horizontal line, to represent the perfect vacuum line; lay off on this line a convenient number of equal distances, and at the points of division erect perpendiculars. The distance between the extreme perpendiculars will represent the stroke of the piston of our hypothetical cylinder, plus its clearance.

To the right of the left hand point, ley off a distance c, to represent the clearance, and erect a perpendicular, upon which lay off the initial total steam pressure per square inch, to a scale of from 10 to 20 pounds to the inch.

The distances from the left hand lower corner of the diagram, to the right, will represent portions of the stroke of the piston, and will be *proportional* to the successive volumes of the steam, when the piston is at the points 1, 2, 3, 4, etc.

These distances will, therefore, be inversely proportional to the corresponding theoretical pressures of the steam.

The extreme length of the diagram should represent the stroke of the piston of our hypothetical cylinder, plus its clearance, or the volume of the expanded steam at the completion of a stroke.

The pressures having been calculated, lay them off on the successive perpendiculars and connect their extremities by a fair curve, which will be the theoretical expansion curve.

This curve may also be constructed according to the method explained in Art. 84, and illustrated in Fig. 44.

To be more explicit, let us suppose that the compound engine

has a piston stroke of three feet, and that the area of the low-pressure piston is *four times* that of the high-pressure piston. Suppose, also, that the clearances of the high and low-pressure cylinders are equivalent to 0.15 of a foot added to the strokes of their pistons, respectively, and that the cut-off is at one-third stroke.

Now, in order that the length of the base line shall represent the stroke of the piston of the hypothetical engine, or the aggregate volume of the low-pressure piston displacement, and the clearances, of both cylinders of the compound engine, its length must be such that, according to the scale adopted for the diagram, it will represent

$$0.15 + 4\,(0.15 + 3) = 12.75 \text{ feet.}$$

There should then be 12 divisions of the base line, each representing a foot of stroke, and being, say, half an inch long. Perpendiculars being erected at the points of division, the points A_1, E_1 and V_1 are to be fixed. The distance $O\,A_1$ will be $0.15 \times 0.5 = 0.75$ of an inch; the distance $O\,E_1$ will be $3.15 \times 0.5 = 1.575$ inches; and the distance $O\,V_1$ will be $(3.15 + 4 \times 0.15) \times 0.5 = 1.875$ inches.

The total steam pressures, taken from the high and low pressure diagrams, may be laid off to a scale of 15 pounds to an inch.

The adiabatic curve, or the curve which represents the diminishing pressures, upon the hypothesis that the steam expands without either giving up or receiving heat, may be constructed by laying off, on the ordinates, pressures which may be calculated by the formula

$$p\,v^{\frac{17}{16}} = p_1\,v_1^{\frac{17}{16}}$$

whence,

$$p_1 = p\left(\frac{v}{v_1}\right)^{\frac{17}{16}}; \quad \dots\dots\dots\dots\dots\dots (59)$$

in which p and p_1 are the pressures corresponding to the volumes v and v_1, respectively.

Examples.—The following example is given as an exercise for the student. The data are from the Gaskill compound engines of the Saratoga water-works.

ENGINE DATA.

High-pressure cylinder, diameter	21	inches.
High-pressure piston-rod, diameter	3	"
High-pressure piston, stroke	36	"
Low-pressure cylinder, diameter	42	"
Low-pressure piston-rod, diameter	3.5	"
Low-pressure piston, stroke	36	"
Clearance, high-pressure cylinder	2.35 %	
Clearance, low-pressure cylinder	2.50 %	

DIAGRAM DATA.

	H. P. Cyl.	*L. P. Cyl.*
Initial pressure, total lbs	85.	23.5
Cut-off, high-pressure cyl., fraction of stroke.	0.275	
Pressure at 0.1 stroke	85.	17.0
" " 0.2 "	85.	14.0
" " 0.275 "	85.	....
" " 0.3 "	84.	12.4
" " 0.4 "	68.	11.0
" " 0.5 "	56.	10.0
" " 0.6 "	48.5	9.0
" " 0.7 "	43.0	8.5
" " 0.8 "	37.5	8.0
" " 0.9 "	34.0	7.5
" " 1.0 "	31.0	7.0

The indicator may also be applied to pumps, both water and air, to air-compressors and to blowing engines, for the purpose of indicating their action.

88. *Effect of Cushioning.*—Heretofore the clearance space has been supposed to be void, at the instant of admission, and that the filling of this space, involved an expenditure of steam, for which only a slight return was realized, in the form of work, during the period of expansion. As a matter of fact, however, the clearance space is always filled with steam, or vapor, of greater or less tension; and it, therefore, follows, that the expenditure of boiler steam, in filling the clearance space, or in raising the tension of the steam therein to that of the boiler steam, is always less than it would be if the space were void.

It is probable that, in engines having a high piston speed, the best condition is that in which, by an early closure of the exhaust valve, the vapor or steam thus confined on the exhaust

side of the piston, is cushioned to such an extent that its tension becomes just equal to that of the boiler steam. Fig. 49.

The expenditure of boiler steam, to fill the clearance space, is thus wholly avoided; but it is avoided at the expense of the work performed in compressing, or in cushioning the steam. The conditions may be so adjusted, that, theoretically, the loss due to clearance, in any given case, will be a minimum. This is done by comparing the work due to the expansion of the clearance steam, with that expended in cushioning the exhaust, and then making the excess of the latter over the former, a minimum. The student will find this an interesting and a valuable exercise.

IX. General Dimensions of Engines.

89. *The Steam Cylinder.*—In order to construct a formula for the diameter of the cylinder, let

$H\text{-}P =$ the indicated horse-power.
$s =$ stroke of the piston, in feet.
$r =$ number of revolutions per minute.
$p_m =$ mean indicated steam pressure, in pounds per square inch.
$d =$ diameter of cylinder, in inches.

Then

$$H\text{-}P = \frac{2 \times 0.7854\, d^2 p_m r s}{33000}$$

$$= \frac{2 \times 0.7854}{33000} \times d^2 p_m r s$$

$$= 0.0000476\, d^2 p_m r s.$$

If the numerical coefficient be represented by c, we have,

$$H\text{-}P = c\, d^2 p_m r s. \quad \ldots\ldots\ldots\ldots (a)$$

Now, put

$$s = \frac{u\, d}{12};$$

u being the ratio of the stroke of the piston to the diameter of the cylinder, and ranging in value, from 0.48 to 2.5.

Substituting this value in (a), we get

$$H\text{-}P = \frac{u\, c\, d^3 p_m r}{12}. \quad \ldots\ldots\ldots\ldots (b)$$

Putting, finally,

$$c' = \frac{c}{12} = 0.000004,$$

for $\frac{c}{12}$, in (b) we get

$$H\text{-}P = u c' d^3 p_m r;$$

whence

$$d = \sqrt[3]{\frac{H\text{-}P}{u c' p_m r}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

The value of u is small, where space is limited, as in screw vessels; in side-wheel vessels, where the cylinders are placed either fore and aft, or vertically, as in beam engines, and on land, where space is not limited, u has larger values.

If a given speed of piston is to be maintained, as v feet per minute, then

$$r = \frac{v}{2s} = \frac{6v}{u d};$$

which, in (b), gives

$$H\text{-}P = \frac{6 u c d^3 p_m v}{12 u d}$$
$$= \frac{c d^2 p_m v}{2};$$

whence

$$d = \sqrt{\frac{2 H\text{-}P}{c p_m v}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (61).$$

Example 1.—Required the diameter of the cylinder and the stroke of the piston of a 100 horse-power engine, in which $u = 2.5$, $p_m = 50$ lbs., and $r = 50$.

Making the proper substitutions in (60) we have,

$$d = \sqrt[3]{\frac{100}{2.5 \times 0.000004 \times 50 \times 50}} = 16\ 1 \text{ inches},$$

say, 16 inches, and slightly increase p_m.

Then, since $u = 2\ 5$, we have for the stroke of the piston,

$$s = 2.5 \times 16 = 40 \text{ inches}.$$

Example 2.—A simple condensing engine is to be constructed, which shall be capable of driving the machinery of a cotton factory having 600 looms, and about 30,000 spindles; steam is to be carried at 75 pounds above the atmosphere, and is to be

expanded $3\frac{1}{3}$ times; the piston speed is to be 360 feet per minute, and a vacuum of 26 inches is to be maintained; $H\text{-}P = 360$.

Required, the diameter of the cylinder, and the stroke of the piston.

First—To find p_m we have, Eq. (36),

$$p_m = \frac{p\,(1 + \log_e 3\frac{1}{3})}{3\frac{1}{3}} - 2$$

$$= \frac{(75 + 14.7)\,(1 + 1.1)}{3\frac{1}{3}} - 2 = 60.79 \text{ pounds;}$$

say, 60 pounds.

Then, substituting in Eq. (61), we have

$$d = \sqrt{\frac{2 \times 360}{0.0000476 \times 60 \times 360}} = 26\,45 \text{ inches;}$$

Say, 27 inches.

If $u = 2.5$, $s = 2.5 \times 27 = 67.5$ inches, say, 66 inches.

The cylinder will therefore be 27×66, and will make

$$r = \frac{360}{2 \times 5.5} = 32.73$$

revolutions per minute.

As this speed is too small, for the best results, let us assume a speed of 720 feet per minute.

Then, since, (61), d varies as

$$\sqrt{\frac{1}{v}},$$

we shall have, for a new diameter,

$$d' = d\sqrt{\frac{360}{720}}$$

$$= 26.45 \times 0.71 = 18.77 \text{ inches;}$$

say, 19 inches, and for the corresponding stroke of piston,

$$s' = 2.5 \times 19 = 47.5 \text{ inches,}$$

say, 48 inches.

Thus we have a cylinder, 19x48, and the engine must make

$$r = \frac{720}{2 \times 4} = 90 \text{ revolutions per minute.}$$

Example 3.—Required the dimensions of the cylinders of a pair of simple condensing engines, for a screw steamship of 3300

tons displacement, which is to develop a speed of 16 knots; the screw-shaft to make 70 revolutions, and the engines 35 revolutions, per minute, with steam 25 pounds above the atmosphere, expanded twice, and a vacuum of 26 inches.

Here, $H\text{-}P = 2400$ horse-power, for each engine, and

$$p_m = \frac{(25 + 14.7)\,(1 + log_e.\,2)}{2}$$

$$= \frac{39.7\,(1 + 0.69)}{2} = 33.55 \text{ pounds;}$$

say, 36 pounds, and let the steam be carried a little higher. so that the increase above 25 lbs., together with the 0.6 lb. above, will cover the imperfect vacuum.

Let, also, the speed of piston be $v = 280$ feet per minute, and $u = 0.5$.

Then, Eq. (61),

$$d = \sqrt{\frac{2 \times 2400}{0.0000476 \times 36 \times 280}} = 100 \text{ inches, nearly,}$$

and

$$s = 0.5 \times 100 = 50 \text{ inches.}$$

The cylinders were made 100×48 inches, and the speed was 16.8 knots; probably with a steam pressure materially greater than 25 pounds.

In the case of a compound engine, in which there is to be an assumed measure of expansion, in the high-pressure cylinder, and in both cylinders, determine, first, the dimensions of a single cylinder which will, theoretically, develop the required power, with the given steam pressure, and with the total measure of expansion; the resulting dimensions will be those of the low-pressure cylinder.

Then, second, if, e be the measure of expansion in the high-pressure cylinder, and E be the measure of expansion in both cylinders, the *area* (a), of the high-pressure piston will be

$$a = A.\,\frac{e}{E};$$

in which A is the area of the low-pressure piston.

Or, since the areas of the pistons are proportional to the squares of their diameters, if d and D be the diameters of the high and low-pressure pistons, respectively, we may write

$$d^2 = D^2 . \frac{e}{E};$$

whence

$$d = D\sqrt{\frac{e}{E}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

90. ***Relation of Clearance to Stroke of Piston.***—In Fig. 50, there are represented two cylinders of equal diameters, but of different lengths; the clearances, supposed to be the same in both, are also supposed to be the smallest that are practicable for that particular diameter of cylinder. It is evident that the clearance in cylinder a, constitutes a smaller percentage of the piston displacement than it does in cylinder b, and, therefore, that the loss due to clearance, other things being equal, is less in a long-stroke engine than it is in a short-stroke engine.

This conclusion may also be reached as follows:

Since

$$\frac{c}{s} < \frac{c}{s'},$$

it follows, that for a cut-off at a given fraction of the piston stroke, in each of two cylinders, of unequal lengths, the period of expansion will be longer, absolutely, in the longer cylinder than it will be in the shorter one. The measure of expansion, of all the steam,—and therefore of the clearance steam—in the longer cylinder will be

$$\frac{s + c}{a + c};$$

while the measure of expansion of all the steam in the shorter cylinder will be

$$\frac{s' + c}{a' + c}.$$

Now, since $\frac{s}{a} = \frac{s'}{a'}$, and since c is always very small, relatively to both, it follows that

$$\frac{s + c}{a + c} > \frac{s' + c}{a' + c} \text{*}$$

Therefore the work due to the expansion of a given volume of clearance steam is greater in a long stroke engine than it is

*Let $c = 1$; $a = 4$; $s = 16$; $a' = 10$ and $s' = 40$.

Then $\frac{4 + 1}{16 + 1} > \frac{10 + 1}{40 + 1}$; or, $0.2947 > 0.275$.

in a short stroke engine, and hence the *loss*, due to clearance, is less in the long, than in the short stroke engine.

With the small clearance, however, of modern engines, amounting to no more than 2.5 per cent in cylinders of 36 inch stroke of piston, the difference in losses due to clearance, as well as the losses themselves, are practically so small that it is scarcely worth while to undertake to estimate them.

91. *Areas of Steam and Exhaust Ports.*—In order that the steam may reach the cylinder promptly, and with very nearly the full tension of the boiler steam, proper capacities must be provided, not only in the steam pipe, but in the opening of the steam port, through which the steam enters the cylinder.

Again, in order that the steam, after doing its work in the cylinder, may escape quickly, and thus relieve the piston from objectionable counter-pressure, the areas of the exhaust openings must be sufficiently large.

The key to the determination of these capacities and areas is in the steam-port, the proper area of which may be determined as follows:

Steam enters the cylinder, through the steam port, in consequence of a difference in pressures, or tensions, of the steam in the steam chest and in the cylinder.

This difference in pressures should not exceed a moderate limit, say 0.3 of a pound.

The velocity with which the steam passes through the steam port, may be regarded as the velocity due to the height of a column of the entering steam, whose weight is equal to the difference in pressures already referred to; the section of the column being, of course, one square inch.

In order to express the conditions in the form of an equation, let

D = the diameter of the piston, in inches.
A = the area of the piston, in square inches.
s = the stroke of the piston, in feet.
r = revolutions of the engine, per minute.
V = the mean velocity of the piston, in feet per second.
a = the area of the steam-port, in square inches.
$k\,a$ = the effective area of the steam-port.
v = the velocity of the entering steam.
p = the pressure of the steam in the steam chest, in pounds per square inch above zero.

δ = the difference between the pressures in the steam-chest and in the cylinder.

h = the height due to v.

C = the specific volume of the steam.

Then,

$$v = \sqrt{2gh},$$

and

$$kav = ka\sqrt{2gh};$$

which represents the volume of steam admitted to the cylinder per second. This volume is also represented by AV.

We have, therefore,

$$ka\sqrt{2gh} = AV,$$

whence,

$$a = \frac{AV}{k\sqrt{2gh}} \dots\dots\dots\dots\dots\dots\dots (a)$$

Now,

$$A = 0.7854\,D^2, \text{ and } V = \frac{2rs}{60} = \frac{rs}{30}.$$

These, in (a), give

$$a = \frac{0.7854\,D^2 rs}{30k\sqrt{2gh}} \dots\dots\dots\dots\dots\dots\dots (b)$$

In order to determine h, in terms of C and δ, we have but to remember that a column of steam, C feet high, and having a base one foot square, weighs 62.425 pounds; while if the base be one square *inch*, the weight of the column will be

$$\frac{62.425}{144} = 0.4335 \text{ of a pound.}$$

Then, to find how much of this column will be required to give the weight δ, which represents the difference in pressures, we have

$$0.4335 : \delta :: C : h,$$

whence

$$h = \frac{\delta C}{0.4335} = 2.312\,\delta C;$$

which, in (b), gives

$$a = \frac{0.7854}{30\sqrt{2 \times 32.16 \times 2.312}} \times \frac{D^2 rs}{k\sqrt{\delta C}}, \dots\dots\dots (c)$$

If $\delta = 0.3$, (c) becomes

$$a = 0.004\frac{D^2 rs}{k\sqrt{C}} \dots\dots\dots\dots\dots\dots (63)$$

For long and narrow rectangular ports, k may be taken as 0.8; and in that case (63) becomes

$$a = 0.005 \frac{D^2 r s}{\sqrt{C}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (64)$$

For circular ports, we may make $k = 0.62$, in which case (63) becomes

$$a = 0.00645 \frac{D^2 r s}{\sqrt{C}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

In the foregoing, our volumes have been expressed in *inches* multiplied by feet; the final results are obviously the same as they would have been, had the unit of volume been a *cubic foot.*

Example 1.—If $D = 100$ inches, $r = 40$, $s = 4$ feet, and $p = 60$, (64) gives

$$a = 0.005 \times \frac{100^2 \times 40 \times 4}{\sqrt{428}} = 386.5 \text{ square inches.}$$

These ports may be made 82 inches long; their widths will then be

$$\frac{386.5}{82} = 4.71 \text{ inches.}$$

If the ports be double—in order to secure the admission of the steam without excessive travel of the valve, the width of each of the two *parts* of the port will be $\frac{4.71}{2} = 2.35$ inches.

As designed and built, these ports were as follows:

Steam ports (2) .... 3.5″ × 82″
Exhaust port .... 16″ × 82″

The steam openings, at each end of the cylinder, were, therefore,

$$2 \times 3.5 \times 82 = 574 \text{ square inches,}$$

and the exhaust openings,

$$16 \times 82 = 1312 \text{ square inches;}$$

or more than double the steam openings.

Example 2.—Let $D = 60$ inches, $r = 65$, $s = 2\frac{1}{6}$ feet, and $p = 45$ pounds.

Then, (64) gives

$$a = 0.005 \times \frac{60^2 \times 65 \times 2\frac{1}{6}}{\sqrt{561.5}} = 107.0 \text{ square inches.}$$

If $s = 3$ feet,

$$a = 107.0 \times \frac{18}{13} = 148.1 + \text{ square inches.}$$

If the ports be 54 inches long, their widths will be

$$\frac{148.1}{54} = 2.74 \text{ inches.}$$

As designed and built, the ports were as follows:

Steam ports............................ 2.75″ × 54″,
Exhaust port........................... 5.5″ × 54″;

with steam "lap" to cut-off at $\frac{5}{8}$ stroke.

Example 3.—Let $D = 27$ inches, $s = 3$ feet, $r = 15$, and $p = 90$ pounds.

Then, the ports being circular, (65) gives

$$a = 0.00645 \times \frac{27^2 \times 15 \times 3}{1\ 295} = 12.37 \text{ square inches;}$$

which corresponds to a port about 4 inches in diameter.

As designed and built, there are two circular ports, for a "double beat" valve, 9 inches in diameter; giving an aggregate opening for the admission of steam, which is to the theoretical opening, as

$$2 \times 9^2 : 4^2;$$

or, $$162 : 16;$$

or, $$1 : 0.1.$$

These ports are made large because the valves are to be raised, habitually, very much less than one-fourth their diameters.

The exhaust ports are 10 inches in diameter, with double-beat valves. The area of the exhaust opening, as compared with that of the steam opening, is therefore, as 2×10^2 is to 2×9^2, or as 200 to 162.

In this case, it may be remarked, the difference between the boiler and cylinder pressures is absolutely inappreciable, on a card traced to a scale of 40 lbs. to the inch.

It was for a long time the general practice to make the exhaust ports about twice as large as the steam ports; it being assumed that so large an excess was necessary to insure the free and prompt exit of the steam. This practice is still followed, in some quarters.

It would seem, however, that, if a difference of 0.3 of a pound between the pressures of the steam in the steam chest and in the cylinder is sufficient to *admit* the steam, the difference be-

tween the cylinder pressure and the condenser pressure, which always amounts to several pounds, would be sufficient to discharge the exhaust steam through an opening no larger than the steam port.

Actual practical results, however, are more satisfactory than the results of speculation or theory.

At the "First Millers' Exposition," held at Cincinnati, Ohio, in June, 1880, three of the very best and most efficient modern engines, of their kind, were subjected to a competitive test, which was conducted by Mr. John W. Hill. From Mr. Hill's report, the figures in the following table are extracted:

Name of Engine.	Pressures.			Efficiency.	Areas in Square Inches.		
	Boiler.	Initial *cyl.*	Vacuum inches.		Steam port.	Exhaust port.	Steam port by Eq. (64).
Harris-Corliss	95.09	90.07	25.67	0 876	13.50	24.75	30.95
Reynolds-Corliss...	95.83	91.09	25.45	0.878	15.75	27.00	30.95
Wheelock........	95.25	88.09	23.98	0.909	30.00	30.00	30.95

The cylinders of these engines were all of the same size, and were 18 inches in diameter, by 48 inches stroke of piston, with clearances of 1.9, 2.6 and 2.3 per cent. respectively. The mean piston speed of each was about 600 feet per minute, and the performances were so nearly equal, and so remarkably good, that Mr. Hill found great difficulty in deciding as to their relative merits.

In the Harris-Corliss and in the Reynolds-Corliss engines, it will be observed that the steam ports are only half as large as would be given by Eq. (64); while in the Wheelock engine, the steam and exhaust ports are equal, and agree with the areas given by Eq. (64).

It necessarily follows, then, that since (64) gives results twice as large as those which, in practice, give not only satisfactory, but most excellent, results, they must be ample, if not, indeed, excessive; yet the results of (64) are small, as compared with examples 1 and 2.

It will be observed, too, that the exhaust ports of the Harris-Corliss and of the Reynolds-Corliss engines, which were both smaller than the steam port given by (64), gave, in each case, a better vacuum than that of the Wheelock engine, with an exhaust port which was practically equal to that given by (64).

The smaller vacuum of the Wheelock engine was, however, due, probably, to the special condensing apparatus used.

The difference between the boiler and initial steam pressures, amounting to from five to seven pounds, may have been due to the length and insufficient area of the steam-pipe. The steam-chest pressures are not given.

92. *Diameter of Piston Rod.*—This depends, of course, upon its length, upon the strength of the material, and upon the effective steam pressure on the piston.

A simple formula for the diameter of the piston-rod may be constructed as follows:

Columns of wrought iron, with flat ends, are found to sustain the following loads, without yielding:

When the length is less than 12 diameters, 14,225 pounds per square inch of section; when the length is between 12 and 24 times the diameter, 11,877 pounds; and when the length is greater than 24 diameters, 7,112 pounds.

In practice, one-half of each of these values may be used—it being remembered, of course, that no excessive lengths are required.

If $s =$ the safe load, per square inch, on the rod,
$d =$ the diameter of the rod, in square inches,
$a =$ the area of a transverse section of the rod, in inches, and
$P =$ the safe load, in pounds, on the rod,

then,

$$P = a s = 0.7854 d^2 s.$$

Substituting, now, for s, successively, the three values given above, we get for

$$l < 12 d, \ P = \frac{14225}{2} \times 0.7854 d^2;$$
$$\text{for } l = 12 d, \text{ to } 24 d, \ P = \frac{11877}{2} \times 0.7854 d^2;$$
$$\text{and for } l > 24 d, \ P = \frac{7112}{2} \times 0.7854 d^2.$$

These pressures or loads must be made equal to the maximum effective pressure on the steam piston.

Let

$D =$ the diameter of the steam piston, in inches, and
$p =$ the effective steam pressure, per square inch.

Then,

$$P = 0.7854 D^2 p.$$

Equating this, successively, with the foregoing values of P, we get,

1. $$\frac{14825}{2} \times 0.7854\, d^2 = 0.7854\, D^2 p;$$

whence

$$d = 0.011\, D \sqrt{p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (66^a)$$

2. $$\frac{11877}{2} \times 0.7854\, d^2 = 0.7854\, D^2 p;$$

whence

$$d = 0.013\, D \sqrt{p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (66^b)$$

3. $$\frac{7112}{2} \times 0.7854\, d^2 = 0.7854\, D^2 p;$$

whence

$$d = 0.017\, D \sqrt{p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (66^c)$$

Steel Rods.—The relative abilities of iron and steel to resist compression, without yielding, may be taken at 1745 to 2518. The transverse sections of rods of the two materials, of the same length, and of equal strength, should, therefore, be made inversely proportional to these numbers; or, their diameters should be made inversely proportional to the square roots of these numbers.

Now,

$$\sqrt{\frac{1745}{2518}} = 0.83.$$

In order, then, to determine the proper diameters of steel rods, for each of the relations of l to d, we must multiply the numerical coefficients of (66^a), (66^b) and (66^c) by 0.83.

We thus get:

1. For the case where l is less than 12 diameters,

$$d = 0.83 \times 0.011\, D \sqrt{p}$$
$$= 0.009\, D \sqrt{p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (67^a)$$

2. For the case where l is between 12 and 24 diameters,

$$d = 0.83 \times 0.013\, D \sqrt{p}$$
$$= 0.011\, D \sqrt{p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (67^b)$$

3. For the case where l is greater than 24 diameters,

$$d = 0.83 \times 0.017\, D \sqrt{p}$$
$$= 0.014\, D \sqrt{p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (67^c)$$

In order to determine, approximately, the relation between l and d, take for iron rods, $d = \frac{1}{8} D$, and for steel rods, $d = \frac{1}{9} D$; then use the proper formula.

The formulæ are applicable to the determination of the diameters of ***short***, ***medium*** and ***long*** piston rods of both materials.

Example 1.—Let $D = 27$ inches; $p = 75 + 13 = 88$ pounds, and the stroke $= 36$ inches.

Then, approximately, $d = \frac{27}{8} = 3\frac{3}{8}$ inches; which is so near one-twelfth of 36 inches that we will use (66^b) or (67^b).

For an iron rod, therefore.

$$d = 0.013 \times 27 \sqrt{88} = 3.3 \text{ inches;}$$

say, 3.5 inches.

Example 2.—Let $D = 25$ inches, $l = 12\, d$ nearly, and $p = 90$ pounds.

Then, for an iron rod, we have, (66^b),

$$d = 0.013 \times 25 \times \sqrt{90} = 3.0875 \text{ inches.}$$

The actual diameter is 3.11 inches.

Example 3.—In the Harris-Corliss engine, tested at the Cincinnati Exposition, to which reference has already been made,

$D = 18$ inches, $p = 95 + 13 = 108$ pounds, and the stroke $=$ 48 inches. The rods were probably of steel.

The approximate diameter of the piston-rod is $\frac{18}{9} = 2$ inches; which is exactly $\frac{1}{24}$ of the stroke. As this is the dividing line between lengths greater and less than 24 diameters, we use (67^c) and get,

$$d = 0.014 \times 18 \times \sqrt{108} = 2.62 \text{ inches.}$$

This would be the same for each of the other two engines.

The actual diameters of the piston-rods of the three engines were as follows:

Of the Harris-Corliss..................	2.68	inches.
Of the Reynolds-Corliss	2.81	inches.
Of the Wheelock......................	2.9375	inches.

Example 4.—In the Allis engines of the Allegheny City waterworks, the diameters of the high-pressure pistons is $D = 31$ inches; the pressure $p = 108 + 13 = 121$ pounds, and the stroke is 48 inches.

The piston-rods being, presumably, of iron, we have for their approximate diameters, $\frac{31}{8} = 3.875$ inches; indicating a length of over 12 diameters. We therefore use (66^b), and get,

$$d = 0.013 \times 31 \times \sqrt{121} = 4.43 \text{ inches.}$$

The actual diameters are $5\frac{5}{8}$ inches; showing a large excess of material and strength in the rods.

Example 5.—In the Gaskill pumping-engines, at Saratoga, N. Y., the high-pressure pistons are 21 inches in diameter, and the stroke of the piston is 36 inches; the steam pressure $p = 75 + 13 = 88$ pounds, and d is between 12 and 24 diameters. Using (66^b) we get,

$$d = 0.013 \times 21 \times \sqrt{88} = 2.56 \text{ inches.}$$

The actual diameters of the high-pressure piston-rods were 3 inches.

It will be observed that even the best practice is variable; and that, while three first-class engines of Example 3, had piston-rods which were very nearly in accord with the results given by our formulæ, all the other engines had rods materially larger than those given by the formulæ. It will be safer, perhaps, in each case, to increase, slightly, the results given by (66) and (67); or, to increase the numerical coefficients of each of the six formulæ, (66^a) to (67^c), both inclusive, by about 10%.

93. *The Connecting-Rod.*—Fig. 51. The length of the connecting-rod is made from 1.5 to three times the stroke of the piston. Its diameter, at the necks, *n n*, is made about the same as that of the piston-rod; while from the necks toward the middle of the rod, the diameter is increased at about the rate of $\frac{1}{4}$ inch to the foot.

When the connecting-rod is quite long, it is trussed, in the plane of its vibration, as shown in Fig. 52.

The brasses are in two pieces, as represented at *B B*, Fig. 51, and they are "set-up" or adjusted to the journals, by the gib, *g*, and the key, *k*.

Fig. 53 illustrates an excellent method of adjusting the brasses, which has been adopted in some modern engines.

The tapering key, *k*, is adjusted and held in place by the two screws *s s*.

The adjustment of the brasses is a most important and delicate one, and one which is essential to the proper working of the engine.

If the brasses be set up too tightly, the journals are liable to heat, even when properly lubricated; when they are slack, pounding and rapid wear result, from the lost motion, or back-lash, which is thus permitted.

The method illustrated in Fig. 53 admits of a much more delicate adjustment than is practicable with the gib and the key of Fig. 51.*

94. *Diameter of Shaft-Journals.*—In considering the proper diameter of the main shaft journals, in any case, it is necessary, first, to determine the positions of the cranks in which the maximum torsional stresses occur.

There may be three cases.

1. That of the single engine, with its single crank. In this case the maximum torsional stress occurs when the crank is perpendicular to the connecting-rod.

In this position the thrust or tension upon the connecting-rod, or the force acting upon the crank-pin, at right angles to the crank-arm, is equal to the effective pressure on the steam piston divided by the cosine of the angle between the axis of the connecting-rod and the axis of the cylinder, produced.

This cosine will always be nearly unity, and for this reason we may treat the connecting-rod as parallel to the axis of the cylinder, and consider the thrust or tension upon it as equal to the gross effective pressure on the piston. The slight error, resulting from this assumption, will always be on the side of safety.

2. In the case of two engines, connected at right angles, or where the two cranks are placed 90° apart, on the shaft, the position of maximum torsional stress on the journals is determined as follows:

In Fig. 54, let

P = the force acting on one crank-pin.

P' = the force acting on the other crank-pin.

a = crank-arm, or distance between shaft and crank-pin centres.

u = the sum of the moments of P and P' about the axis of the shaft.

As the result will be entirely independent of the values of P, P' and a, we may consider each of them as equal to unity.

*The details of construction, and of adjustment, are only mentioned incidentally, and in a very general way, for want of time and space. General and essential elements of design and construction can alone be considered; but these will be supplemented, where it is important and practicable to do so, by personal suggestions and explanations.

Then

$$u = \sin.\ \theta + \sin.\ (\theta + \alpha)$$
$$= \sin.\ \theta + \sin.\ \alpha \cos.\ \theta + \cos.\ \alpha \sin.\ \theta,$$

and

$$\frac{du}{d\theta} = \cos.\ \theta - \sin.\ \alpha \sin.\ \theta + \cos.\ \alpha \cos.\ \theta = 0;$$

or,

$$1 - \sin.\ \alpha \tan.\ \theta + \cos.\ \alpha = 0;$$

whence,

$$\tan.\ \theta = \frac{1 + \cos.\ \alpha}{\sin.\ \alpha}.$$

But, since $\alpha = 90°$, $\sin.\ \alpha = 1$, and $\cos.\ \alpha = 0$; these, substituted in the above, give

$$\tan.\ \theta = \frac{1 + 0}{1} = 1;$$

$$\therefore \theta = 45°.$$

It appears, therefore, that the position of the cranks which insures the maximum torsional stress on the journals, is that in which they make angles of 45° with the axes of their respective cylinders.

3. In the case of three engines, connected at angles of 120°, or where the cranks are distributed at equal angular intervals of 120°, each, about the axis of the shaft.

In Fig. 54, consider the forces P', P'' and P''', and the crank-arm a, as being each equal to unity; we then have, for the sum of the moments of the forces about the axis of the shaft,

$$u = \sin.\ \theta + \sin.\ (\alpha + \theta) + \sin.\ \left(\theta + \frac{\alpha}{2}\right);$$

$$= \sin.\ \theta + \sin.\ \alpha \cos.\ \theta + \cos.\ \alpha \sin.\ \theta + \sin.\ \theta \cos.\ \frac{\alpha}{2} + \cos.\ \theta \sin.\ \frac{\alpha}{2};$$

whence,

$$\frac{du}{d\theta} = \cos.\ \theta - \sin.\ \alpha \sin.\ \theta + \cos.\ \alpha \cos.\ \theta + \cos.\ \theta \cos.\ \frac{\alpha}{2} - \sin.\ \theta \sin.\ \frac{\alpha}{2}$$

$$= 1 - \frac{\sin.\ \alpha \sin.\ \theta}{\cos.\ \theta} + \cos.\ \alpha + \cos.\ \frac{\alpha}{2} - \sin.\ \frac{\alpha}{2}\ \frac{\sin.\ \theta}{\cos.\ \theta},$$

$$= 1 - \sin.\ \alpha \tan.\ \theta + \cos.\ \alpha + \cos.\ \frac{\alpha}{2} - \sin.\ \frac{\alpha}{2} \tan.\ \theta = 0;$$

whence,

$$tan.\,\theta\left(sin.\,\alpha + sin.\,\frac{\alpha}{2}\right) = 1 + cos.\,\alpha + cos.\,\frac{\alpha}{2},$$

and

$$tan.\,\theta = \frac{1 + cos.\,\alpha + cos.\,\frac{\alpha}{2}}{sin.\,\alpha + sin.\,\frac{\alpha}{2}}. \quad \ldots\ldots\ldots\ldots\ldots (a)$$

But, since $\alpha = 120°$, $cos.\,\alpha = -\,cos.\,60° = -\,cos.\,\frac{\alpha}{2}$.

Also, $sin.\,\alpha = sin.\,120° = sin.\,60° = sin.\,\frac{\alpha}{2}$.

Substituting these values in (a), we get,

$$tan.\,\theta = \frac{1 + cos.\,\frac{\alpha}{2} - cos.\,\frac{\alpha}{2}}{2\,sin.\,\frac{\alpha}{2}}$$

$$= \frac{1 + cos.\,60° - cos.\,60°}{2\,sin.\,60°} = \frac{1}{2 \times \frac{1}{2}\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

But $\frac{1}{\sqrt{3}} = tan.\,30°$,

$$\therefore \theta = 30°.$$

It therefore follows, that the position of maximum torsional stress on the shaft journals, is that in which one crank is at right angles to the axis of its cylinder, and in which each of the other two cranks will make an angle of 30° with the axis of its cylinder.

Of course, the position of one crank determines the positions of the other two—the angles between the cranks being known.

The resistance to torsional stress, of a cylindrical rod, varies as the cube of its diameter, and the moment of this resistance may be expressed by $m\,d^3$; in which d is the diameter, in inches, and m, the moment of the breaking load, in foot-pounds, of a journal one inch in diameter.

"A square bar of wrought iron, having a journal one inch in diameter and one-quarter of an inch long, *twisted* with a load of 326 pounds, and *broke* with a load of 570 pounds, applied at the extremity of a lever arm of 2.5 feet."

Now, 570 pounds upon an arm of 2.5 feet is equivalent to

$2.5 \times 570 = 1425$ pounds, upon an arm of one foot. The moment of the ultimate resistance of the experimental shaft was therefore, 1425, and we may write, for any journal, d inches in diameter,

$$1425\, d^3.$$

For safety, we may use one-seventh of this value; or, say $200\, d^3$.

If

P = maximum pressure on the crank-pin in pounds and

a = the length of the crank-arm, in feet,

we may write

$$P a = 200\, d^3. \qquad \dots\dots\dots\dots\dots\dots (68)$$

To express P and a in terms of the diameter of the cylinder, stroke of the piston and steam pressure, let

D = diameter of the steam cylinder, in inches,
s = stroke of the piston, in feet, and
p = maximum pressure, per square inch, on the piston, or the pressure is indicated by the gauge, plus the additional pressure due to the vacuum.

Then

$$P = 0.7854\, D^2 p,$$

and, for a *single* engine, $a = \dfrac{s}{2}$.

$$\therefore P a = \frac{0.7854}{2} \times D^2 p s;$$

which, in (68), gives

$$d^3 = \frac{0.7854}{2 \times 200} \times D^2 p s;$$

whence, finally,

$$d = \sqrt[3]{0.002\, D^2 p s}. \qquad \dots\dots\dots\dots\dots (69)$$

For *two* engines, we have

$$P' a' = 200\, d^3,$$

in which, since there are two cylinders, and two cranks, we shall have

$$P' = 2 \times 0.7854\, D^2 p,$$

and

$$a' = \frac{s}{2} \times sin. 45^\circ = \frac{0.7\, s}{2},$$

$$\therefore P' a' = \frac{2 \times 0.7 \times 0.7854}{2} \times D^2 p\, s = 200\, d^3;$$

from which we get

$$d = \sqrt[3]{0.0027\, D^2 p\, s} \quad \dots\dots\dots\dots\dots\dots (70)$$

For *three* engines, we have

$$200\, d^3 = P\, a + P\, a\, sin. 30^\circ + P\, a\, sin. 30^\circ$$

$$= \frac{P\, s}{2} (1 + 2\, sin. 30^\circ)$$

$$= P\, s.$$

But

$$P\, s = 200\, d^3 = 0.7854\, D^2 p\, s;$$

hence

$$d^3 = \frac{0.7854}{200} \times D^2 p\, s,$$

and

$$d = \sqrt[3]{0.0039\, D^2 p\, s} \quad \dots\dots\dots\dots\dots\dots (71)$$

95. *Comparison of Results.*—If d_1, d_2 and d_3 be the diameters of the shafts in three cases, respectively, and D, p and s be constant, we have, from (69), (70) and (71),

$$d_1 : d_2 : d_3 :: \sqrt[3]{20} : \sqrt[3]{27} : \sqrt[3]{39}$$

$$:: 1 : 1.1 : 1.25;$$

whence it appears that the shafts of *single*, *double* and *triple* engines, having cylinders of the same capacity, and using the same steam pressure, are proportional to 1, 1.1 and 1.25, respectively.

For very long shafts, somewhat larger diameters are required in order to prevent excessive torsion, under stress.

Example 1.—In the case of double engines, $D = 70$ inches, $s = 10$ feet, and $p = 30 + 13 = 43$ pounds.

Then, (70),

$$d = \sqrt[3]{0.0027 \times 70^2 \times 43 \times 10} = 17.85 \text{ inches};$$

say, 18 inches.

These data are from the U. S. Frigate Susquehanna, whose shaft was 18 inches in diameter.

This shaft broke, in the side of the ship, at Fortress Monroe, in smooth water, in June, 1861.

At that time, the boilers being old, the steam-pressure was rarely carried above 17 pounds in them; adding the vacuum, we get $17 + 13 = 30$ pounds, for which a diameter of 16 inches would have been ample.

The break was due to imperfect forging.

Ordinarily, of course, in a side-wheel vessel, the main journals are subject to stress from only a single engine; but in a sea-way, with one wheel nearly or wholly out of water, the journals on the opposite side are subjected to the action of both engines.

Example 2.—In the U. S. S. Wampanoag and other ships of her class, there were double engines, in which $D = 100$ inches, $s = 4$ feet, and $p = 47 + 13 = 60$ pounds. These, in (70), give

$$d = \sqrt[3]{0.0027 \times 100^2 \times 60 \times 4} = 18.64 \text{ inches.}$$

The journals were 18 inches in diameter.

Example 3.—The U. S. Frigate Niagara had three engines, in which $D = 72$ inches, $s = 3$ feet, and $p = 20 + 13 = 33$ pounds. These, in (71), give

$$d = \sqrt[3]{0.0039 \times 72^2 \times 33 \times 3} = 12.6 \text{ inches.}$$

The engine journals were 14, $15\frac{1}{4}$ and 17 inches, respectively, and the main shaft journals $16\frac{1}{2}$ inches.

In this case, with the low steam-pressure used, 12-inch journals would probably have been ample; but, in comparison with the large engines, they would have appeared to be too small.

On this vessel, steam was only designed to be used as auxiliary to the sails.

It is suggested that the student, when traveling by steamer, take occasion to collect the necessary data, and substitute them in (69), (70) or (71), according as there may be one, two or three engines. He will thus be able to compare the practice of different engine builders with the results given by the formulæ.

96. *Cylinder of Minimum Condensation.*—The cylinder which will radiate the minimum quantity of heat, is that in which, other things being equal, the surface is a minimum; or it is the cylinder which, for a given capacity, has the minimum surface.

In order to determine the relation between the length and diameter of such a cylinder, let

$l =$ the length of the cylinder,
$d =$ its diameter,
$V =$ its volume, and
$S =$ its surface.
Then,

$$S = \pi d l + \frac{\pi d^2}{2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

and

$$V = \frac{\pi d^2 l}{4}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

From (b), we get

$$\pi l = \frac{4V}{d^2}; \text{ or } \pi d l = 4 V d^{-1};$$

which, in (a), gives

$$S = 4 V d^{-1} + \frac{\pi d^2}{2}.$$

Differentiating, and placing the first differential coefficient equal to zero, we have

$$\frac{dS}{dd} = -4 V d^{-2} + \pi d = 0;$$

whence

$$d = \sqrt[3]{\frac{4V}{\pi}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (72)$$

This value of d, substituted in (b), gives

$$l = \sqrt[3]{\frac{4V}{\pi}} = d;$$

which, since the second differential coefficient is positive, is a minimum.

Thus, it appears that the cylinder of minimum condensation, from radiation, has its diameter equal to its length.

With small cylinders, and high piston speeds, the pressure and temperature of the steam being constant, powers will be developed which, with low-piston speeds, would require larger cylinders.

High-piston speeds, are, therefore, favorable to the economical development of power.

The gain, due to high-piston speeds, resulting as it does from the diminished radiation from a smaller cylinder surface, is only

moderate; for the reason that it is due to a saving, in only a small part of the heat expended, while the steam is doing its work.

The moderate gain will, however, be proportional to the increase in piston speed.

In order to demonstrate this, let us suppose the piston speed to be doubled.

Let

$d =$ the diameter, and stroke, in a given case.
$P =$ the gross pressure on the piston.
$P_1 =$ the gross pressure on the smaller piston.
$x =$ the diameter, and stroke of piston, of the cylinder which, with the same steam pressure and measure of expansion, and with twice the piston speed, will maintain an equivalent power.
$n =$ the number of revolutions, per minute, of the smaller engine.

Then, since the piston speed is to be doubled, we shall have $P_1 = \frac{1}{2}P$; whence it follows, that the area of the smaller piston must be just one-half that of the larger piston.

The diameter, and stroke, x, will then be equal to $d\sqrt{\frac{1}{2}}$.

The surface of the cylinder whose diameter and piston stroke $= d$, is

$$\frac{\pi d^2}{2} + \pi d^2 = \tfrac{3}{2}\pi d.$$

The surface of the cylinder whose diameter, and piston stroke $= x$, is

$$\frac{\pi x^2}{2} + \pi x^2 = \tfrac{3}{2}\pi x^2 = \tfrac{3}{2}\pi \times \tfrac{1}{2}d^2 = \tfrac{3}{4}\pi d^2.$$

This last surface is just one-half that of the cylinder whose diameter is d.

It follows, therefore, that the radiating surface of the smaller cylinder is one-half that of the larger one.

If, now, the loss of heat, by radiation, be proportional to the radiating surface, the loss from the smaller cylinder will be only one-half that from the larger one.

It will have been noticed that the stroke of the piston has, in each case, been taken as representing the length of the cylinder; and it has been upon this hypothesis than the demonstration has been effected.

As a matter of fact, however, the length of the smaller cylinder is relatively larger, when compared with its diameter, than is the length of the larger cylinder, when compared with *its* diameter; and it necessarily follows, that the radiating surface of the smaller cylinder is slightly *greater* than half that of the larger one, and, therefore, that the gain in economy, due to the higher piston speed and smaller cylinder, while it is very nearly proportional to the piston speed, is not precisely so.

While the advantage resulting from high piston speeds must be conceded, it must be remembered that high speeds involve more perfect and more expensive workmanship, more delicate adjustment, and greater care and skill in supervision; and that the damage, resulting from neglect and breakage, is liable to be far more serious than it is where lower speeds are employed.

97. *Thickness of the Cylinder.*—The necessary thickness of the metal of the steam cylinder depends less upon the tendency of the steam to rupture it than upon other considerations. *Rigidity* is the essential requisite, and experience is, in this case, the only safe guide.

In practice, the thickness ranges from 0.75 of an inch to about 2 inches. A safe rule is to make

$$t = 0.75 + \frac{d}{100} \text{ inches; } \ldots\ldots\ldots\ldots\ldots\ldots (73)$$

in which d is the diameter of the cylinder, in inches, and t its thickness.

98. *The Fly-Wheel.*—The discussion of this subject belongs, properly, to the course in Mechanics, and for this reason we only refer to it here in a general way.

The regulating effect of the fly-wheel, depends upon the weight and diameter of its rim, and upon its velocity.

When the pressure of the steam in the cylinder is greater than the load upon the engine, the excess is expended in increasing the velocity of the fly-wheel, and thus storing up work in it, to be given out again when the relations between the load and steam pressure are reversed.

While the fly-wheel acts as a regulator of velocity, it can only do so through a change in its velocity, and in that of the engine. This change in velocity will be greater or less, as the weight and diameter of the fly-wheel are less or greater. In practice, it ranges from 0.2 to 0.01 of the mean velocity.

If W be the weight of the rim of the fly-wheel, and the velocity of its centre of gravity be increased from v_1 to v_2, the work w, which will be stored up during the change will be

$$w = W\left(\frac{v_1^2 - v_2^2}{2g}\right). \quad \ldots\ldots\ldots\ldots\ldots(74)$$

Now, if Fig. 56 represent a mean indicator card from the cylinder of a single engine, and the line $m\,m$ represent the mean pressure line, per square inch, the works to be stored up in the fly-wheel, during the first part of the stroke of the piston, and given out by it during the latter part of the stroke, will be represented by the equivalent shaded areas, A and B. The measure of these works will be found by multiplying the areas, in foot-pounds, by the number of square inches in the surface of the piston.

Then, assuming the diameter of the wheel, or the diameter of the circular path described by the centre of gravity of its rim, and fixing the permissible change in the velocity, the values of v_1 and v_2 will be readily found.

For example, let the diameter of the path described by the centre of gravity of the fly-wheel rim be 10 feet, and the number of revolutions, 50 per minute.

The mean velocity of the rim will, therefore, be

$$\frac{10 \times 3.1416 \times 50}{60} = 26.18$$

feet per second.

Let, now, the permissible change in velocity be one foot per second.

Then

$$v_1 = 26.18 + 0.5 = 26.68,$$

and

$$v_2 = 26.18 - 0.5 = 25.68$$

feet per second.

Finally, w having been determined from the indicator diagram and area of the piston, we substitute in (74), and get

$$w = W\left(\frac{26.68^2 - 25.68^2}{2 \times 32.16}\right);$$

whence

$$W = \frac{2 \times 32.16\,w}{26.68^2 - 25.68^2} = 1.23\,w. \text{ pounds.}$$

If the shaded area A, Fig. 56, be $25 \times 2 = 50$ foot pounds, and the area of the steam piston be 200 square inches, we shall have $w = 50 \times 200 = 10{,}000$ foot-pounds, and

$$W = 1.23 \times 10{,}000 = 12{,}300 \text{ pounds.}$$

If the engine be double, construct a diagram which shall represent the combined action of the steam on a square inch of each piston, during a complete stroke of one piston and the corresponding *parts* of a stroke of the other piston. Such a diagram is represented in 57, in which $m\ m$ is the line of mean combined pressure on a square inch of each piston, during a complete stroke of one, and the corresponding parts of the other.

In this case, the requisite regulating effect, and weight, of the fly-wheel are less than before, because the areas A and B are smaller; but, it will be observed, there are twice as many changes in speed as before.

In a three-cylinder engine, the requisite regulating effect will be still smaller, with three times as many changes in speed, per stroke, as occur in the single engine.

In a quadruplex engine, the regulation required is smaller still, with four changes of speed during a stroke.

In Fig. 57, A_1 and B_1, represent the works, below the mean pressure line, during the periods of the stroke when the steam-pressures are greater and less than the mean pressure, respectively; P_i is the initial pressure, P_f the final pressure, and P_m the mean pressure.

In an investigation of different types of pumping engine, four cases were considered, with reference to probable regularity of action, simply; the first was a single cylinder, direct acting engine, in which the steam was supposed to be cut-off at 0.4 of the stroke; the second was a double engine, connected at right angles, of the same capacity, and cutting-off at the same fraction of the stroke; the third was a quadruplex non-compound engine, of like capacity, with two engines connected to each crank, and with one crank set 135° ahead of the other—the two engines connected to each crank being so placed that the axes made an angle of 90°; the fourth was the same as the third, but was operated as a compound engine, the steam following half-stroke in one cylinder, and being exhausted into the other three cylinders.

In each case, the steam followed 0.4 of the stroke, and in the

last two cases, it will be observed, the cranks and engines were so placed that steam was admitted to, and exhausted from, some cylinder at four equal intervals during each stroke of one of the pistons.

Diagrams were made, in each case, representing the gross pressure, on all of the pistons, at each tenth of a stroke of one of the pistons, and the values of A, B, A_1, B_1, P_1, P_f and P_m were determined. These values, with several important ratios, are given in the following table:

SUMMARY OF RATIOS.

Type.	$\frac{A}{A_1}$	$\frac{B}{B_1}$	$\frac{P_1 - P_m}{P_m}$	$\frac{P_m - P_f}{P_m}$	$\frac{P_1 - P_f}{P_1}$
1.	0.2842	0.3141	0.3268	0.5170	0.6360
2.	0.0822	0.0822	0.1857	0.2357	0.3550
3.	0.0427	0.0639	0.0924	0.1180	0.1920
4.	0.2095	0.2694	0.3456	0.4784	0.6120

It will be noted that the ratios for the fourth type are in nearly every case practically the same as those for the first type. This fact indicates that both types require practically the same regulations; in other words, the quadruplex compound engine requires a fly-wheel equal to that required by the single engine of the same capacity; while the quadruplex non-compound engine (3), as indicated by the several ratios, requires a much smaller fly-wheel than either of the other three types.

As a result of the investigation, the quadruplex non-compound engine was adopted, in preference to either of the others, and for the reasons stated.

99. *The Crank.*—The proportions of cranks are best determined by certain arbitrary practical rules, representing good practice. In Fig. 58, having fixed the positions of the centres of the crank-pin and shaft, draw circles, with these points as centres, having diameters equal to the diameters of the crank-pin and shaft, respectively; next, draw circles concentric with these, and having diameters 1.8 times the diameters of the crank-pin and shaft; we thus have front elevations of the hub and eye of the crank.

To construct a front elevation of the web, which connects the hub and eye, erect perpendiculars to the axis $c\ c$, at the centres

of the shaft and crank-pin, and on these lay off on either side of *c c* distances equal to 0.7 of the diameters of the shaft and crank-pin, respectively, and connect the points thus fixed, on each side of the axis; the connecting lines, *c c* and *c′ c′*, will be the outlines of the front elevation of the web.

The depths of the hub and eye are made equal to the diameters of the shaft and crank-pin respectively, and the depth of the web *b*, is so fixed as to be in harmony with the other dimensions of the crank.

A crank, thus designed, will have ample strength, and will present a more satisfactory appearance than one in the design of which all the refinements of mathematical and mechanical science have been brought into requisition.

Cranks are made of wrought iron or steel, and sometimes, though rarely, of cast iron, The eye and hub are bored a trifle smaller than the crank-pin and shaft are turned, and both are sometimes given a slight taper; the eye and hub are heated and expanded, after which the crank is placed on the shaft, and the pin in the eye; the eye and hub shrink in cooling, and grip the crank-pin and shaft, to which they are further secured by keys or otherwise.

Care is to be taken that the difference between the diameters of the shaft and the hub is not so great that, in cooling, the hub will be strained to the point of rupture.

The moment of the resistance to shearing, of the key, should be made equal to the moment of the force acting at the crank-pin.

The length and diameter of the crank-pin, especially in large engines, are determined rather by the area of the projection of that part of the pin which receives the pressure, than by considering its strength. Experience teaches that the pressure on the piston should not exceed 500 pounds per square inch of the projected area of the crank-pin. The pressure on the piston being known, the projected area of the crank-pin should, therefore, be made equal to that pressure divided by 500, in square inches.

100. *Length of Working Beam.*—In beam engines the length of the working beam, between centres, should be such that the paths described by the end centres will diverge equally on each side of the axis of the steam-cylinder, produced, and a parallel line drawn through the centre of the main shaft, respectively. In other words, the axis of the cylinder, produced, should bisect

the versed sine $v\ v$, Fig. 59, of the arc described by the corresponding end centre of the beam, and the parallel, through the main shaft centre, should bisect the versed sine of the arc described by the crank-end centre of the beam.

Let

$2d =$ the distance between the axis of the cylinder and the centre of the main shaft.

$2l =$ the length of the beam, between centres.

$s =$ the stroke of the piston.

$\theta =$ the angle, at the beam centre, between the two extreme positions of a line joining the end centres of the beam.

$2v =$ the versed sine of the arc described by each end centre.

Then,

$$l = d + v. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Now,

$$(d + v)^2 - \left(\frac{s}{2}\right)^2 = (d - v)^2;$$

or,

$$d^2 + 2\,dv + v^2 - \frac{s^2}{4} = d^2 - 2\,dv + v^2;$$

whence,

$$4\,dv = \frac{s^2}{4}$$

$$\therefore v = \frac{s^2}{16d}.$$

Substituting this in (a), we get,

$$l = d + \frac{s^2}{16d}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

The working beam is usually made of cast iron, and strengthened by a strap of wrought iron, shrunk on, giving it the familiar form shown in Fig. 60.

X. Power Required by Steam Vessels.

101. *Construction of a Formula for Power.*—It was formerly the practice of mechanical engineers to base their estimates of the power, required by a steam vessel, upon the area of the mid-ship section, upon the desired speed, and upon a constant, depending upon the form of the immersed body of the vessel.

Let

$A =$ the area of the mid-ship section, in square feet,
$V =$ the desired speed, in knots, per hour,
$C =$ a constant, depending upon the model.

Then

$$H\text{-}P = C A V^3. \qquad (76)$$

In fixing the value of C, to be used in any case, the engineer relied upon his experience and judgment, and the results were, as a rule, not altogether satisfactory.

The expression, it will be observed, is equivalent to

$$C \times A V^2 \times V.$$

It was thus assumed that the resistance varied as the squire of the speed, and the power, as the product of the resistance by the space through which the vessel moved.

An examination of this formula discloses the following defects:

First—Only the *net* power varies as the cube of the speed. This part of the power constitutes only about 60 to 66* per cent. of the total power. The remaining 34 to 40 per cent. is made up of the power expended in overcoming the friction of the engine, *per-se*, and the power expended in overcoming the back-pressure, both of which vary as the speed simply; of the power expended in overcoming the friction due to the load on the engine, which varies nearly as the cube of the speed, and of the power expended in the slip of the screw or paddle-wheel, which varies nearly as the cube of the speed. To these should be added the power expended in overcoming the friction between the screw-blades and the water, which Chief Engineer Isherwood, U. S. N., estimates as equivalent to 0.45 of a pound for each square foot of surface, having a velocity of 10 feet per second, and which is equivalent to 0.0045 of a pound per square foot, with a velocity of one foot per second.

Recent experiments upon the friction of water in large water pipes seem to indicate that this coefficient should be about 0.006, or 33 per cent. larger than the value used by Chief Engineer Isherwood.

Second—Too much stress seems to have been laid upon the model of the vessel. Within the limits of ordinary practice, it is believed that the effect of slight changes in the sharpness or

*These percentages are somewhat too small.

fullness of the lines is very moderate, and that the resistance opposed by the water, depends rather upon the relations between the displacement and the *wet surface* of the immersed body; indeed, it is beginning to be recognized, that the minimum resistance is secured by giving the immersed body such proportions as will render the wet surface, for a given displacement, a minimum.

In order to construct a more satisfactory formula for the requisite power, let

D = the displacement of the vessel, in tons of 2000 pounds.

l = extreme length of the vessel, on the water line, in feet.

C_1 = a constant, depending upon the power P_1, expended in overcoming the back and friction pressure.

C_2 = a constant, depending upon the power P_2, usefully applied to the shaft, together with the friction due to that power.

Then, since P_1 varies as the speed, and assuming that it also varies as the mid-ship section of the immersed body, we have

$$P_1 = C_1'AV. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Similarly,

$$P_2 = C_2'AV^3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

Now, in order to express A in terms of D, we have, from the relations between similar surfaces, and volumes, and their homologous dimensions,

$$A \propto l^2; \therefore l \propto A^{\frac{1}{2}};$$

also,

$$D \propto l^3; \therefore l \propto D^{\frac{1}{3}};$$

whence

$$A^{\frac{1}{2}} \propto D^{\frac{1}{3}}, \text{ or } A \propto D^{\frac{2}{3}}$$

In any case, then, the mid-ship section may, within the limits of good practice, be regarded as proportional to the $\frac{2}{3}$rd power of the displacement, either in cubic feet or in tons.

Writing, now, in (a) and (b), $D^{\frac{2}{3}}$ for A, and C_1 and C_2, for C_1' and C_2', respectively, we get

$$P_1 = C_1 D^{\frac{2}{3}} V, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a')$$

and

$$P_2 = C_2 D^{\frac{2}{3}} V^3. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b)$$

The sum of these two parts of the total power will be the total power, or

$$
\begin{aligned}
H\text{-}P &= P_1 + P_2 \\
&= C_1 D^{\frac{2}{3}} V + C_2 D^{\frac{2}{3}} V^3 \\
&= D^{\frac{2}{3}} V (C_1 + C_2 V^2) \quad \ldots\ldots\ldots\ldots (77)
\end{aligned}
$$

To find the values of C_1 and C_2, we have, from (a') and (b'),

$$C_1 = \frac{P_1}{D^{\frac{2}{3}} V} \text{ and } C_2 = \frac{P_2}{D^{\frac{2}{3}} V^3}, \quad \ldots\ldots\ldots\ldots (c)$$

respectively. Take, now, the cases of any number of vessels actually built, whose displacements, speeds and corresponding powers are known, and determine the proper distribution of power, Art. 82, in each case; thus we find the values of P_1 and P_2 for each vessel.

Substitute the values of $D^{\frac{2}{3}}$, V and P_1 in the first of the above expressions, and find C_1; substitute the values of $D^{\frac{2}{3}}$, V^3 and P_2 in the other, and find C_2.

The distribution of the power, in the cases of five U. S. steam frigates, gave values of P_1 and P_2, which, when substituted in the two expressions of (c), gave the values of C_1 and C_2, which are contained in the following table:

Name of Vessel.	C_1	C_2
Merrimack	0.18184	0.0047345
Wabash	0.14847	0.0045799
Minnesota	0.15875	0.0045725
Roanoke	0.13619	0.0048133
Niagara	0.15067	0.0047227
Mean values	0.1552	0.0046846

These mean values, of C_1 and C_2, substituted in (77), give, finally,

$$H\text{-}P. = D^{\frac{2}{3}} V (0.1552 + 0.0046846 V^2). \quad \ldots\ldots\ldots (78)$$

The first four of the vessels named had full lines; but the fifth had, for her time, fine lines; and yet there appears no corresponding difference in the values of C_1 and C_2.

Equation (78) may be written as follows:

$$H\text{-}P = C D^{\frac{2}{3}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (79)$$

in which the coefficient C, is

$$C = V(0.1552 + 0.0046846\ V^2).$$

Substituting in this expression different values of V, from 10 knots to 20 knots, both inclusive, we get the several values of C in the following table:

V Knots per Hour.	C	V Knots per Hour.	C
10	6.2366	16	21.6713
11	7.9424	17	25.6538
12	9.9574	18	30.1140
13	12.3097	19	35.0862
14	15.0273	20	40.5808
15	18.1385		

Example.—Required, the total power of the engines which will give a vessel of 1000 tons displacement a speed of 14 knots per hour.

$$\begin{aligned} H\text{-}P &= 1000^{\frac{2}{3}} \times 15.0273 \\ &= 100 \times 15.0273 \\ &= 1502.73. \end{aligned}$$

Upon trial, the speed of the vessel was 14 knots, and the power, 1522 horses.

The displacement of a vessel, in tons, is expressed as follows:

$$D = k\frac{l\,b\,d}{35}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

in which l = the length, on water line, in feet; b = breadth of beam, in feet; d = mean draught of water, in feet; 35 = volume of a ton of sea water in cubic feet, and k = a coefficient, expressing the ratio of the volume of the immersed body to the volume of the circumscribing parallelopipedon. k ranges, in value, from 0.4 to 0.6.

102. *English Examples and Precedents.*—

London Engineering, Nov. 22, 1878, *pp.* 417–419, says: "In relation to the trial of H. M. S. Inflexible, that old idea that the mid-ship section was a measure of the power required, is shown to be wholly fallacious, and the tendency now is, to *more beam* and finer ends, as offering less resistance, for a given displacement.

"The old Admiralty formulæ were:

$$Speed^3 = \frac{constant \times I.\ H\text{-}P.^*}{D^{\frac{2}{3}}};$$

also,

$$Speed^3 = \frac{constant \times I.\ H\text{-}P}{mid\text{-}ship\ section}.$$

A *mean* of the two results was adopted.

"Mr. Froude, by his investigations and experiments has shown, first, that skin friction is a large element of resistance; and, finally, that mid-ship section has no claim to a place in speed calculations."

The Inflexible, then recently tried, has, absolutely and relatively, the largest beam of any vessel in Her Majesty's service.

On March 19th, 1876, the following table was copied—except the last two columns, the first of which contains the results given by (78), for the several displacements and powers, and the second the speeds made on the trial trips after completion. The vessels were then building.

ENGLISH IRON-CLADS.

Name of Vessel.	*D* Tons.	*I. H-P.*	Speeds, Knots.	
			By Eq. (78)	Trial Speeds
Inflexible	11165	8000	14.25	14.75
Dreadnaught	10950	8000	14.4	.....
Thunderer	9190	5690	13.25	.....
Temeraire	8412	7000	14.6	14.65
Nelson	7323	6000	14.3	.....
Alexandria	9492	8000	14.9	15.00
Shannon	5103	3500	12.8	.....

On the trial trips of the Temeraire, Alexandria and Inflexible, the displacements, indicated powers and speeds, together with the speeds deduced from (78), were as follows:

Name.	*D* Tons.	*I. H-P.*	Speeds—Knots.	
			Trial.	Calculated.
Temeraire	8571	7516	14.65	14.89
Alexandria	9432	8616	15.00	15.35
Inflexible	9500	8407	14.75	15.18

The trial of the Inflexible lasted 6 hours, during which the steam-pressure was 60 pounds, and the coal consumption, per indicated horse-power per hour, 2.05 pounds.

**I. H-P.* = indicated horse-power.

The following table contains the dimensions, displacements, powers and speeds, both trial and calculated, of several British war vessels :

Data and Results.	Hercules.	Sultan.	Temer-aire.	Alexan-dria.	Inflexible.
Length, between perpendiculars	325′	325′	285′	325′	324′
Breadth, extreme	59′	59′	62′	63′-8″	75′
Mean draught, on trial..........	24′-8¼″	24′10¾″	27′	26′-1½″	20′-11″
Displacement, tons.............	8696	8714	8581	9432	9500
Indicated horse-power:.........	8529	8620	7516	8615	8407
Speed, knots	14.69	14.13	14.65	15.	14.95
Speed, calculated	15.52	15.6	14 60	14.9	14.25

In all of the calculations, the indicated horse-powers were used, as the total powers, which were slightly greater, were not known.

103. *Miscellaneous Examples.*—There are now* under contract, in England, for the Japanese government, two war vessels which are designed to make 18.5 knots. The displacements and powers, substituted in (78), give speeds of 18.5 knots. In other words, the formula used by the designers of the machinery for these vessels, and (78) give, in this case, identical results.

Absolutely accurate results, as to speed or power, are not, of course, to be expected.

2. In the *Journal* of the Franklin Institute, for January or February, 1882, will be found the results of a trial of the

U. S. S. DISPATCH.

In this case,

$l = 174$ feet; $b = 25.5$ feet; draught, aft, $= 15,33$ feet; mean draught, 12.2 feet.

The displacement was 574 tons, the wet surface, 5576 square feet, and

$$k = \frac{D}{bdl} = 0,425.$$

This, it will be remembered, is the ratio of the volume of the displacement, to that of the circumscribing parallelopipedon.

The engine was double, with cylinders 33⅛ inches and 34¼ inches diameter, respectively, and 33 inches stroke of piston.

*January, 1886.

The result of the trial was as follows:

Total *H-P*	493.75
Indicated *H-P*	402.79
Net *H-P*	370.42
Speed, knots	10.75
Slip of screw, per cent	15.09

The power given by (78), for these conditions, is 517.2 horses, which exceeds the actual power by 4 per cent.

We have here the means of testing the theory that the net power is almost wholly due to the resistance of skin friction, inasmuch as we have given the wet surface, the speed in knots, and the net power.

Let,

a = the area of the wet surface, in square feet.

v = the speed, in feet per second.

f = the resistance, per square foot of wet surface, due to a velocity of one foot, per second, and

$H\text{-}P_n$ = the net horse power required.

Then,

$$H\text{-}P_n = \frac{a\,f\,v^3}{550}; \quad \dots\dots\dots\dots (81)$$

in which

$$v = \frac{6086 \times 10.75}{3600} = 18.173$$

feet per second, and $f = 0.006$ of a pound (See Hydraulic Notes). Substituting these, and the value of $a = 5576$ square feet, in (81), we get

$$H\text{-}P_n = \frac{5576 \times 0.006 \times 18.173^3}{550} = 362.074.$$

This result, it will be observed, is only 8.346 horse-power, or 2.25 per cent., less than the actual net horse-power expended during the trial.

Every opportunity for testing this method of estimating the net power should be improved. The total power will be found, approximately, by multiplying the net power by a coefficient varying from 1.33 to 1.66—for engines of steam vessels, only.

3. A steam vessel is 335.5 feet long on the load water line, with a beam of 45 feet and a load draught of 18.5 feet; required the total power which will give the vessel a speed of 16 knots.

Assume k, Art. 101, to be 0.55; then we shall have

$$D = \frac{335.5 \times 45 \times 18.5}{35} \times 0.55 = 4382.5,$$

say, 4400 tons, and (79) will give

$$H\text{-}P = D^{\frac{2}{3}}C$$
$$= 4400^{\frac{2}{3}} \times 21.67 = 5819.$$

Let the diameter of the screw be 18 feet, the minimum pitch be 27 feet ($= 1.5\ d$) and the maximum pitch 31 feet. Let, also, the slip of the screw be 15 per cent. of the minimum pitch. The advance of the vessel, per revolution of the screw, will then be.

$$27\ (1 - 0.15) = 23 \text{ feet};$$

and the number of revolutions of the screw, per minute,

$$\frac{16 \times 101.43}{23} = 70.56.$$

If the engines be geared to the screw shaft, in the ratio of 84 to 41, the revolutions of the engines, per minute, will be

$$\frac{41 \times 70.56}{84} = 34.44$$

If there be two engines, in which the ratio of stroke of piston to diameter of Cylinder (u) is 0.5, and the speed of the piston, 275 feet per minute, the steam pressure, 35 pounds above the atmosphere, and the cut-off, at two-thirds of the stroke, we shall have for the total power of each engine,

$$\frac{5819}{2} = 2909.5,$$

say, 2910 horses.

Then, (35^b),

$$p_m = (35 + 14.7)\left(\frac{1 + log_e.\,1.5}{1.5}\right)$$
$$= 49.7 \times \frac{1 + 0.405}{1.5} = 46.57 \text{ pounds},$$

say, 46.5 pounds per square inch; and, (61),

$$d = \sqrt{\frac{2 \times 2910}{0.0000476 \times 46.5 \times 275}} = 98.6 \text{ inches}.$$

The stroke of the piston, $u\ d$, will be

$$0.5 \times 98.6 = 49.3 \text{ inches}.$$

As actually constructed, the steam cylinders were 100 inches in diameter, with a stroke of piston of 48 inches.

The vessel (U. S. S. Wampanoag), on her trial trip, had the following dimensions and gave the following results:

Length, on water line, feet	355.50
Beam, feet	45.17
Mean, draught, feet	18.50
Displacement, tons	4251.50
Ratio of immersed body to circumscribing parallelopipedon	0.53
Mean steam pressure, pounds	32.42
Mean vacuum, inches *Hg*	24.
Mean revolutions of engine, per minute	31 265
Mean speed, for 36 hours, knots	16.758
Mean revolutions of screw, per minute	64.04

The precise proportions of the screw are not at hand; but they did not differ much from those assumed in the calculations.

4. An iron side-wheel steamer built by the "Iron Steamboat Co.," New York, 1881, has the following dimensions: Length 210 feet, beam 32 feet and draught, about 7.5 feet. Making $k = 0.6$, we have the displacement, in tons,

$$D = \frac{210 \times 32 \times 7.5}{35} \times 0.6 = 864.$$

Let us put the speed at 16 knots = 18.4 miles per hour.

Then, (79),

$$H \cdot P = 864^{\frac{2}{3}} \times 21.6713 = 1966 \text{ horses}.$$

The paddle wheels were 36 feet 9 inches in diameter, by 9 feet face, with shaft journals 14.75 inches in diameter.

If we take the diameter of the centre of pressure of the paddles at 29.25 feet, the circumference of this diameter will be 91.87, say, 92 feet. If we also take the slip of the paddles at 25 per cent., we have the advance of the vessel, per revolution of its wheels,

$$92\,(1 - 0.25) = 69 \text{ feet}.$$

The number of revolutions per minute will therefore be

$$\frac{16 \times 101.43}{69} = 23.5.$$

Make $u = 2.5\ p = 45$ pounds above the atmosphere, and assume the cut-off to be at half-stroke.

Then,

$$p_m = (45 + 14.7)\left(\frac{1 + log_e 2}{2}\right)$$

$$= 59.7 \times 0.85 = 50 + \text{pounds};$$

and, (60),

$$d = \sqrt[3]{\frac{1966}{2.5 \times 0.000004 \times 50 \times 23.5}} = 55.1 \text{ inches,}$$

say, 54 inches = 4 feet 6 inches.

The stroke of the piston,

$$u\,d = 2.5 \times 54 = 135 \text{ inches,}$$

or 11 feet 3 inches.

As constructed, the steam cylinder is 52 inches in diameter, and the stroke of piston, 11 feet.

The boilers were designed to carry 50 pounds of steam. If the vacuum be taken at 26 inches, the indicated initial pressure will be 50 + 13 = 63 pounds per square inch, and (69) will give, for the diameter of the shaft journals,

$$d = \sqrt[3]{0.002 \times 52^2 \times 63 \times 11} = 15.34 \text{ inches,}$$

which is 15 34 – 14.75 = 0.59 of an inch greater than the actual diameter.

We have no facts as to the performance of this steamer on her trial trip.

5. The new Cunarder, Servia, 1881, has the following dimensions: Length, 530 feet; beam, 52 feet; depth of hold, 49 feet 9 inches; probable draught, 26 feet, and $H\text{-}P = 10{,}500$.

Making $k = 0.6$, we get

$$D = \frac{530 \times 52 + 26}{35} \times 0.535 = 10960 \text{ tons.}$$

Now, since, (79),

$$H\text{-}P = D^{\frac{2}{3}}\,C = 10500,$$

we have

$$C = \frac{10500}{10960^{\frac{2}{3}}} = 21.278.$$

Referring to the table, Art. 101, we find that this value of C corresponds to a speed of about 16 knots per hour.

The actual speed of the Servia is not known. Her shortest passage, however, was 7 days, or 188 hours.

Her average speed must have been about 16 knots.

6. The new U. S. Steamer Chicago, which was launched Dec. 5th, 1885, has the following dimensions :

Length, between perpendiculars, feet	315
Beam, feet	$48\frac{1}{6}$
Draught, mean, feet	19
Displacement, tons	4500
Horse-power of engines	5000
Speed expected, knots	15

This vessel is to be provided with two compound beam engines, and with twin screws 15 feet 6 inches in diameter, having a mean pitch of 24 feet 6 inches, and being designed to make 75 revolutions per minute.

The dimensions give, for the displacement of the circumscribing parallelopipedon,

$$D' = \frac{315 \times 48\frac{1}{6} \times 19}{35} = 7894.5 \text{ tons.}$$

The ratio k, is, therefore,

$$k = \frac{D}{D'} = \frac{4500}{7894.5} = 0.57.$$

The horse-power required to make a speed of 15 knots per hour is, (79),

$$H\text{-}P = 4500^{\frac{2}{3}} \times 18.1385 = 4944 \text{ horses};$$

or within 56 horses of the power to be provided.

The mean pitch of the screws being 24.5 feet, if we take the slip as equal to 15 per cent., the advance of the vessel, per revolution of the screws, will be

$$24.5\,(1 - 0.15) = 20.825 \text{ feet.}$$

The advance of the vessel, per minute, will therefore be $75 \times 20.825 = 1561.875$ feet.

As one knot per hour is equivalent to 101.43 feet per minute, it follows that, if our assumption of slip is correct, the speed of Chicago will be

$$\frac{1561.875}{101.43} = 15.394 \text{ knots.}$$

All things considered, therefore, it is reasonably safe to predict that, when the Chicago is completed, and her trial trip is made, the results will be satisfactory to her designers.*

*For results of the Chicago's trial trip see Appendix.

The distribution of the indicated power, Art. 82, in the case of the three engines tested by Mr. Hill, at the Cincinnati Millers' Exposition, was as follows:

Horse-power.	Harris-Corliss.	Reynolds-Corliss	Wheelock.
Friction of Engine	5.80	6.30	4.60
Friction due to load	3.80	3.75	3.80
Power to work air-pump	2.83	2.10	0.40
Net effective horse-power	87.57	87.85	91.20
Indicated horse-power	100.00	100.00	100.00

XI. The Screw Propeller.

104. *Description.*—The modern screw propeller consists of two, three or four blades, attached to a central hub, which is keyed to the rear end of the shaft, under the stern of the vessel. The acting surfaces of the blades are helicoidal, and may be conceived to be generated as follows: About the lower extremity of a vertical line, let two, three or four other lines, each perpendicular to the first, be distributed in such a manner as to make equal, horizontal, angular intervals. Then suppose the two, three or four lines to rise, along the vertical line, at a uniform rate, and at the same time to revolve about the vertical line, at a uniform rate; the surfaces generated by the revolving lines will be the helicoidal surfaces of the two, three or four blades of a screw propeller.

If the revolving lines make a complete revolution about the vertical line, the advance of the revolving lines along the axial line, during the revolution, will be the "*pitch*" of the screw, and the screw will be a true screw.

If, however, the revolving lines advance along the axial line at a uniformly increasing or decreasing rate, while they revolve at a uniform rate, the screw surfaces generated will be those of screws having uniformly expanding or diminishing pitches.

In practice, the revolving lines which generate the screw surfaces, make only a fraction of a complete revolution about the axial line. The angle δ through which they revolve, is such that if n be the number of lines, or the number of screw blades whose surfaces are to be generated.

$$n\delta = \text{from } \tfrac{1}{3} \text{ to } \tfrac{4}{10} \text{ of } 360°.$$

The angle δ will therefore, generally, vary between the limits,

$$\delta = \frac{120^\circ}{n} \text{ and } \delta = \frac{144^\circ}{n}. \qquad \text{(82)}$$

If $n = 2$, $\delta = 60^\circ$ to 72°; if $n = 3$, $\delta = 40^\circ$ to 48°; if $n = 4$, $\delta = 30^\circ$ to 36°.

Again, if the helicoidal surfaces of all the blades of the screw be projected on a plane perpendicular to its axis, or perpendicular to the axial line, the aggregate area, of all the projections, will range between 0.333 and 0.4 of the area of the circumscribing circle.

The foregoing is illustrated in Fig. 61, which represents the projection of a two bladed screw, on a plane perpendicular to its axis.

The length of the screw, measured along the axial line, evidently bears the same relation to the pitch that the angle δ does to 360°; or, if $l =$ the length of the screw and $p =$ the pitch, both in feet, we have

$$l : p :: \delta : 360^\circ$$

$$\therefore \; l = \frac{p\,\delta}{360^\circ}. \qquad \text{(83)}$$

The length is, of course, independent of the number of blades, as the latter are all disposed about a hub, whose length is only slightly greater than l, and as the generating lines all lie in the same plane, perpendicular to the axis.

The angle of the screw, θ, is the angle which the path of the outer extremity of the generating line, at any point, makes with a plane, perpendicular to the axis, passing through that point.

In a true screw, the angle θ is constant; but in a screw of uniformly expanding pitch, the tangent of θ increases, uniformly, as the generating line revolves about the axis of the screw. Conversely, if the screw have a uniformly diminishing pitch, tangent θ will diminish uniformly.

If the path described by the outer extremity of the generating line, during a complete revolution about its axis, be developed upon a plane parallel to the axial line, and passing through the initial point, the development will be either a straight line, a curved line convex upward, or a curved line concave upward, according to the character of the pitch of the screw. If straight, it will be the hypothenuse of a right triangle, whose base is the

development of the circle formed by the horizontal projection of the path, and whose altitude is the pitch of the screw.

Thus, in Fig. 62, $A\ C$ is the development of the spiral path, in the case of the true screw. $A\ e\ C$ is the development, in the case of a screw having a uniformly expanding pitch, and $A\ d\ C$ is the development in the case of a screw having a uniformly diminishing pitch.

$A\ B$ is the development of the horizontal projection of the spiral path, and $B\ C$ is the pitch, in the case of the true screw, and the *mean* pitch, in the case of the screw which has an expanding or diminishing pitch.

We thus have,

$$tan.\ \theta = \frac{p}{circumference}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (83)$$

In sweeping up the acting surface of the screw in the mold-loft, this surface necessarily forms the top of the nowel of the mold, and the posterior of the screw is at the bottom. The generating line, or sweep, therefore generates the acting surface of the blades from the posterior edge, where the pitch is generally a maximum, upward, to the anterior edge, where the pitch is generally a minimum. The surface thus swept up, is therefore that of a screw of uniformly diminishing pitch from the posterior to the anterior edge of the blade. The figure $A\ B\ C\ d$, then, if constructed of metal or of wood, and bent into a circular form, with a diameter equal to that of the screw, will constitute a *guide* for sweeping up the helicoidal surfaces of the blades. The curve $A\ d\ C$, designated the "guide curve," will then occupy the position of the spiral path described by the outer extremity of one of the generating lines of the screw.

This is the practical method of sweeping up the screw. Only a part, however, of the figure $A\ B\ C\ d$ is used; and the base, $A\ b$, of that part, $A\ b\ c$, bears the same relation to the whole circumference $A\ B$, that the angle δ, Fig. 61, bears to 360°; or

$$A\ b : A\ B :: \delta : 360°;$$

whence

$$A\ b = A\ B \frac{\delta}{360°}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

The guide-curve $A\ c$ should be constructed by means of calculated ordinates from the base $A\ b$.

105. *Construction of Screw Formulæ.*—A formula for the determination of the lengths of these ordinates may be constructed as follows:

Suppose the guide-plate, Fig. 63, constructed, and let $A\ B\ C$ represent its development, upon a plane parallel to the axis of the screw. $A\ C$ will be the part of the circumference of the screw occupied by the periphery of a single blade, and $B\ C$ will be the length of the screw, or the terminal ordinate of the developed helix $A\ B$.

Divide $A\ C$ into n equal parts, erect ordinates at each of the points of division, and conceive tangent lines drawn to the curve passing through their extremities, at the extremity of each ordinate. The angles which these tangent lines make with $A\ C$, at their respective points of tangency, will be the angles of the screw at those points.

In Fig. 63, let

$\theta =$ the angle of the screw, at its posterior edge A.
$\theta' =$ the angle of the screw, at its anterior edge B.
$\varphi =$ the pitch of the screw at its posterior edge.
$\varphi' =$ the pitch of the screw at its anterior edge.
$n =$ the number of ordinates.
$x =$ one of the equal divisions of $A\ C = \frac{AC}{n}$.
$y_1 y_2 y_3$, etc., $=$ ordinates to the points a, b, c, etc., of the developed helix $A\ B$.
$R =$ the radius of the screw.
$c =$ the circumference of the screw.
$k =$ a decimal expressing the part of the circumference occupied by one blade of the screw.

If $\varDelta$ be taken to represent the slight decrease in pitch, between A and a, there will be a like decrease in the pitch between a and b, b and c, etc., to B. We shall therefore have,

$$\varphi = \text{pitch at } A,$$
$$\varphi - \varDelta = \text{" \quad " } a,$$
$$\varphi - 2\varDelta = \text{" \quad " } b,$$

and

$$\varphi - n\varDelta = \varphi' = \text{pitch at } B.$$

Making θ_1, θ_2, θ_3, etc., the angles of the screw, at the points a, b, c, etc., we have, (83),

$$tan.\ \theta = \frac{\varphi}{2\,\pi\,R} = \frac{\varphi}{c},$$

$$tan.\ \theta_1 = \frac{\varphi - \Delta}{c},$$

$$tan.\ \theta_2 = \frac{\varphi - 2\Delta}{c},$$

$$tan.\ \theta_3 = \frac{\varphi - 3\Delta}{c},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

and

$$tan.\ \theta_n = tan.\ \theta' = \frac{\varphi - n\Delta}{c}.$$

Taking now the successive differences in the values of the tangents, we have

$$tan.\ \theta - tan.\ \theta_1 = \frac{\varphi}{c} - \frac{\varphi - \Delta}{c} = \frac{\Delta}{c};$$

$$tan.\ \theta_1 - tan.\ \theta_2 = \frac{\varphi - \Delta}{c} - \frac{\delta - 2\Delta}{c} = \frac{\Delta}{c};$$

$$tan.\ \theta_2 - tan.\ \theta_3 = \frac{\varphi - 2\Delta}{c} - \frac{\varphi - 3\Delta}{c} = \frac{\Delta}{c};$$

and so on for the remaining differences. It follows, therefore, that, as has already been stated, the tangents of the screw angles, at successive points, from rear to front, vary by a constant difference, or uniformly, from the after to the forward edge of the blade.

Now, put the constant difference, in the values of the successive tangents, equal to δ_t; then

$$\delta_t = \frac{tan.\ \theta - tan.\ \theta'}{n},$$

$$= \frac{\frac{\varphi}{c} - \frac{\varphi'}{c}}{n} = \frac{\varphi - \varphi'}{c\,n}; \quad \cdots\cdots\cdots\cdots(a)$$

and if α_1 be the angle $a\ A\ C$, between $A\ C$ and the chord $A\ a$,

$$y_1 = x\ tan.\ \alpha_1. \quad \cdots\cdots\cdots\cdots\cdots\cdots(b)$$

But $\alpha_1 = \theta - f\ A\ a$, and since $f\ A\ a$ is the angle included between the tangent to the curve at A, and the chord $A\ a$, we may,

without material error, treat its tangent as equal to the difference between *tan.* θ and *tan.* α_1; or, as equal to half the common difference between the tangents of the angles which the consecutive tangent lines, at A, a, b, etc., make with B.

Now, this last difference being δ_t, we have

$$tan.\ \theta - tan.\ \alpha_1 = \frac{\delta_t}{2};$$

or,

$$tan.\ \alpha_1 = tan.\ \theta - \frac{\delta_t}{2}.$$

This last value, substituted in (b), gives

$$y_1 = x\left(tan.\ \theta - \frac{\delta_t}{2}\right). \quad \ldots\ldots\ldots\ldots\ldots\ldots (c)$$

But,

$$x = \frac{A\,C}{n} = \frac{k\,c}{n},$$

$$tan.\ \theta = \frac{\varphi}{c}, \text{ and } \delta_t = \frac{\varphi - \varphi'}{c\,n}$$

These, substituted in (c), give

$$y_1 = \frac{k\,c}{n}\left(\frac{\varphi}{c} - \frac{\frac{1}{2}\varphi - \frac{1}{2}\varphi'}{c\,n}\right)$$

$$= \frac{k\,c}{n}\left(\frac{\varphi\,n - \frac{1}{2}\varphi + \frac{1}{2}\varphi'}{c\,n}\right)$$

$$= \frac{k}{n^2}\left[\varphi\,(n - \tfrac{1}{2}) + \tfrac{1}{2}\varphi'\right]. \quad \ldots\ldots\ldots\ldots\ldots (d)$$

Again,

$$y_2 = y_1 + x\ tan.\ \alpha_2.$$

But

$$\alpha_2 = \alpha_1 - d\,a\,b;$$

and $d\,a\,b$, being the angle between two consecutive chords, $A\,a$ produced and $a\,b$, and its tangent being equal to the tangent of the angle made by the two corresponding consecutive tangents, we have,

$$tan.\ d\,a\,b = tan.\ \theta_1 - tan.\ \alpha_1 = \delta_t;$$

which, substituted in the expression for y_2, gives

$$y_2 = y_1 + x\,(tan.\ \alpha_1 - \delta_t)$$
$$= y_1 + x\,(tan.\ \theta - \tfrac{3}{2}\delta_t).$$

Substituting, as before, the values of x, tan. θ and δ_1, and putting for y_1 its value in (d), we get

$$y_2 = \frac{k}{n^2}[\varphi(n - \tfrac{1}{2}) + \tfrac{1}{2}\varphi'] + \frac{kc}{n}\left(\frac{\varphi}{c} - \frac{\tfrac{3}{2}\varphi - \tfrac{3}{2}\varphi'}{nc}\right)$$

$$= \frac{k}{n^2}[\varphi(n - \tfrac{1}{2}) + \tfrac{1}{2}\varphi'] + \frac{k}{n_2}[\varphi(n - \tfrac{3}{2}) + \tfrac{3}{2}\varphi']$$

$$= \frac{k}{n^2}[\varphi(2n - \tfrac{4}{2}) + \tfrac{4}{2}\varphi']. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (e)$$

In the same manner, and for the same reasons, we get

$$y_3 = y_2 + x\,(tan.\ \theta - \tfrac{5}{2}\delta_1).$$

Making the proper substitutions, this becomes

$$y_3 = \frac{k}{n^2}[\varphi(2n - \tfrac{4}{2}) + \tfrac{4}{2}\varphi'] + \frac{kc}{n}\left(\frac{\varphi}{c} - \frac{\tfrac{5}{2}\varphi - \tfrac{5}{2}\varphi'}{nc}\right),$$

which reduces to

$$y_3 = \frac{k}{n^2}[\varphi(3n - \tfrac{9}{2}) + \tfrac{9}{2}\varphi']. \quad \ldots\ldots\ldots\ldots\ldots\ldots (f)$$

Similarly,

$$y_4 = \frac{k}{n^2}[\varphi(4n - \tfrac{16}{2}) + \tfrac{16}{2}\varphi')], \quad \ldots\ldots\ldots\ldots\ldots (g)$$

and

$$y_5 = \frac{k}{n^2}[\varphi(5n - \tfrac{25}{2}) + \tfrac{25}{2}\varphi'. \quad \ldots\ldots\ldots\ldots\ldots\ldots (h)$$

Examining, now, the foregoing expressions, we discover the law according to which the values of y_1, y_2, y_3, etc., increase, and may at once write a general formula for any ordinate, in any case.

It appears that the coefficient of n, in the parenthesis, is in each case the number of the ordinate, counting from the beginning of the guide curve; also, that the numerators of the fractions, in the several cases, are equal to the *squares* of the numbers of the ordinates, counting from the beginning of the curve.

If then, m be the number of any particular ordinate, we have, generally,

$$y_m = \frac{k}{n^2}\left[\varphi\left(mn - \frac{m^2}{2}\right) + \frac{m^2\varphi'}{2}\right]. \quad \ldots\ldots\ldots (84)$$

If $m = n$, (84) becomes

$$y_n = \frac{k}{n^2}\left[\varphi\left(n^2 - \frac{n^2}{2}\right) + \frac{n^2\varphi'}{2}\right]$$

$$= \frac{k n^2}{n^2}\left(\frac{\varphi + \varphi'}{2}\right)$$

$$= k\left(\frac{\varphi + \varphi'}{2}\right) = l. \quad \ldots\ldots\ldots\ldots\ldots\ldots(85)$$

If, again, we make $k = l$, or if we extend the blade entirely around the axis, (85) reduces to

$$y_n = \frac{\varphi + \varphi'}{2} = \text{mean pitch.}$$

It will be observed that each ordinate, computed in accordance with (84), is entirely independent of all of the other ordinates; and therefore, that its value will be affected by neither cumulative numerical errors, nor by errors of construction. There is, therefore, no reason why the guide curve, when carefully constructed in the pattern-shop, should not be such as will give a practical result in exact accord with the design of the engineer.

Since the radius of the screw is not involved in the general formula, (84), it follows that a series of ordinates calculated for the periphery of a screw-blade, answers equally well for guide curves at any point between the periphery and the hub. The base $A\,C = k\,c$, will alone vary, from the periphery to the hub.

If it were desirable to construct a screw having a pitch varying from the hub to the periphery, two series of ordinates would be required; one, for a guide curve at the hub, and the other for a guide curve at the periphery.

Two values of φ and two values of c, would also be required.

Example.—Let $\varphi = 24$ feet, $\varphi' = 20$ feet, $k = 0.16$ and $n = 10$.

If the screw is to have two blades,

$$\delta = 0.16 \times 360° = 57.6°,$$

and $2\,\delta = 2 \times 57.6° = 115.2°$. In this case the projected area of the screw surface will be

$$\frac{360}{115.2} = 0.312$$

of that of the circle whose diameter is equal to the diameter of the screw.

The length of the screw will be, (85),

$$l = 0.16\left(\frac{24+20}{2}\right) = 3.52 \text{ feet.}$$

Eq. (84) gives, for the lengths of the ordinates in feet,

		1st Diff's.	2nd Diff's.
$y_1 = \frac{0.16}{100}\left[24\left(10 - \frac{1}{2}\right) + \frac{20}{2}\right]$	$= 0.3808$	0.3808	......
$y_2 = \frac{0.16}{100}\left[24\left(2 \times 10 - \frac{4}{2}\right) + \frac{4 + 20}{2}\right]$	$= 0.7552$	0.3744	0.0064
$y_3 = \frac{0.16}{100}\left[24\left(3 \times 10 - \frac{9}{2}\right) + \frac{9 \times 20}{2}\right]$	$= 1.1232$	0.3680	0.0064
$y_4 = \frac{0.16}{100}\left[24\left(4 \times 10 - \frac{16}{2}\right) + \frac{16 \times 20}{2}\right]$	$= 1.4848$	0.3616	0.0064
$y_5 = \frac{0.16}{100}\left[24\left(5 \times 10 - \frac{25}{2}\right) + \frac{25 \times 20}{2}\right]$	$= 1.8400$	0.3552	0.0064
$y_6 = \frac{0.16}{100}\left[24\left(6 \times 10 - \frac{36}{2}\right) + \frac{36 \times 20}{2}\right]$	$= 2.1888$	0.3468	0.0064
$y_7 = \frac{0.16}{100}\left[24\left(7 \times 10 - \frac{49}{2}\right) + \frac{49 \times 20}{2}\right]$	$= 2.5312$	0.3424	0.0064
$y_8 = \frac{0.16}{100}\left[24\left(8 \times 10 - \frac{64}{2}\right) + \frac{64 \times 20}{2}\right]$	$= 2.8672$	0.3360	0.0064
$y_9 = \frac{0.16}{100}\left[24\left(9 \times 10 - \frac{81}{2}\right) + \frac{81 \times 20}{2}\right]$	$= 3.1968$	0.3296	0.0064
$y_{10} = \frac{0.16}{100}\left[24\left(10 \times 10 - \frac{100}{2}\right) + \frac{100 \times 20}{2}\right]$	$= 3.5200$	0.3232	0.0064

If, in the value of y_{10} we make $k = 1.0$, or substitute 1.0 for 0.16, we get $y_{10} = 22$ feet; which is equal to $\frac{24+20}{2}$; or to the mean pitch. It will be observed, also, that the second differences are all equal. Both of these facts prove the correctness, both of the formula and of the arithmetical operations.

106. *The Working Drawings.*—Fig. 64 represents the guide-plate, as constructed by the pattern-maker, from the calculated ordinates, and other necessary data furnished by the engineer.

Suppose the diameter of the screw to be 16 feet, or $R = 8$

feet. Then draw a centre line KK'; with E as a centre, and with a radius of, say, 8.5 feet*, describe an arc of indefinite extent; do the same with radii of 2, 4, 6 and 8 feet, respectively, and construct the central angle $\delta = 57°.6$, symmetrically, with reference to the axis KK', of the drawing.

Next, divide the radius EK_1 into any number of equal parts, say four, and through the points of division draw the arcs $a\,b$, $c\,d$ and $e\,f$, concentric with FH, and having the same angular extent; then divide the arc AC into ten equal parts (for simplicity only *five* parts are used in the drawing), and from the points of division draw radii to the centre E.

Now, draw tangents to the several concentric arcs, at the points in which they intersect the axis KK', and develop the arcs upon their respective tangents, marking the points of intersection, of the arcs and the radial lines, on the respective developments. The area bounded by the full lines, will be the projection of a blade of the screw upon a plane perpendicular to its axis, and the dotted arc AC, will be the position of the base of the guide-plate, while the acting face of the blade is swept up in the mold.

Next, on the opposite side of the centre E, and at intervals, measured along KK', somewhat greater than $y_n = l$, draw lines perpendicular to KK', and project upon them the developments $F'H'$, $a'\,b'$, $c'\,d'$ and $e'\,f'$, of the concentric arcs FH, $a\,b$, $c\,d$, and $e\,f$ and mark, on the projections, the points corresponding to the intersections of the radial lines and concentric arcs on the projection of the screw-blade.

$F''H''$ will be the projection of $F'H'$, and $a''\,b''$, $c''\,d''$ and $e''\,f''$ will be the projections of $a'\,b'$, $c'\,d'$ and $e'\,f'$, respectively.

On $e''\,f''$ lay off, at its extremities, and at the points of division, the ordinates y_2, y_4, y_6, y_8 and y_{10}, in regular order, as indicated, and draw through their extremities a fair curve. Perpendicular to this curve, at the extremities of the ordinates, lay off other ordinates, representing the thickness of the blade, on the line $e\,f$, and draw through their extremities a fair curve; the space included between these curves, shaded in the figure, is the projection of a developed section of the screw-blade, on the line $e\,f$, at a distance of 2 feet from the centre.

* 8 feet 9 inches would be better, as the space between AC and FH is to be built up in brick. Brick are 8 inches long, and the thinest brick wall is therefore at least 8 inches thick.

In the same manner, lay off the same ordinates on $c'' d''$, $a'' b''$ and $F'' H''$; draw fair curves through their extremities, and on these lay off ordinates representing the thickness of the screw-blade at 4, 6 and 8 feet from the axis, respectively.

Finally draw Fig. 65, showing the sweep, and manner in which it is mounted for use. *H* is a wooden pattern of the hub of the screw, and *M M* is an iron rod or mandril, which is fixed at the axis of the hub, and with it is secured in a vertical position ; the frame work shown is constructed of boards, and constitutes the sweep, which revolves about *M M; F F* is the sweep proper, curved on its under edge, as shown, in order that the blades of the propeller may curve backward, to a centre located at some distance astern, in the axis of the screw produced ; and *A B C* is the guide plate, of plate iron or of wood, depending upon the diameter of the screw.

107. *Molding the Screw.*—With the apparatus shown in Fig. 65, located at a convenient point, and on a level surface, the space included between the guide-plate and the hub, which, in place, is the horizontal projection of the screw-blade, is inclosed by a brick wall ; the inclosed space being then filled in, until the sweep *F F*, as it revolves about *M M* and rises on the guide-curve, is perhaps an inch, at all points, above the filling. The top of the filling is thus, roughly, the form of the screw surface.

Next, molding sand is placed on the filling and is well consolidated; after which its upper surface is swept off to its proper form by the revolving sweep. Fig. 65ᵃ, shows the general appearance of the filling, with its upper helicoidal surface $A\ B\ b'\ a'$, and its bounding brick walls—in dotted lines. This constitutes the "nowel" of the mold.

In the next place the helicoidal surface of the sand has parting sand sifted over it, after which the concentric arcs, of which *a b*, *c d* and *e f* are the projections, are lightly traced. Thickness strips, made either of wood, cut as shown in Fig. 64ᵃ, or of plate metal, cut in the forms shown in the shaded sections of Fig. 64, are bent into form to fit the lines traced on the sand, and are then secured to the sand surface along those lines.

The spaces between these thickness strips are then filled with sand, the upper surface being swept fair with the upper edges of the thickness strips, and then having parting sand sifted over it.

This last mass of sand constitutes the pattern of the screw-blade.

The mass is then built up to a uniform height, somewhat greater than the length of the screw, and covered with a perforated cast-iron plate. In the final building up, first, of a layer of sand, and then of brick, plates of iron are distributed through the mass, and a little above the mass of sand forming the pattern of the blade, each being attached to an iron rod which passes upward and through the top plate where it is secured by a nut or key.

The mold is now complete. After the mold has been properly dried, the upper portion, designated the "*cope*," is lifted off; the sand occupying the position and form of the blade is then removed, after which the cope is replaced.

The same method is pursued for each blade; and when all are complete, the several parts of the mold are placed in their proper positions, the wooden pattern of the hub is removed, a cylindrical core is set at the centre, in the position to be finally occupied by the shaft, and the mold is made ready for receiving the molten metal.

It is not intended to convey the idea that all screws are made in this manner.

Wooden patterns are sometimes made, and sometimes, when a large number of screw propellers, of a given size and pattern, are to be made, metal patterns are used.

The material used is either cast iron, steel, or a composition of copper, tin and zinc, in the proportions 88, 10 and 2, respectively.

The Hirsch patent screw has blades of a peculiar form, in which, while the surface is helicoidal, the sweep is curved in a plane perpendicular to the axis of the propeller. In this screw the blades are detachable; being secured to the hub by very strong bolts.

107. *Slip of the Screw.*—If the medium in which a screw revolves were unyielding, the advance of the screw, per revolution, would be equal to its pitch; but water, being a yielding medium, yields to the action of the screw and recedes, as the screw revolves. If the vessel be secured, as to a dock, the recession of the water will be equal to the pitch of the screw, during each revolution, and the entire power will be expended in overcoming the reaction of the water through a space equal to

the pitch. If, on the contrary, the vessel be free, in open water, it will advance through the water at a speed such that an equilibrium will be established, between the resistance to the motion of the vessel, and the resistance which the receding water opposes to the action of the screw;—which, of course, represents the *thrust* on the screw shaft.

In this case, the advance of the vessel, during a revolution of the screw, will be less than the pitch of the screw; and the difference between the two will be the "slip" of the screw.

If φ be the pitch of the screw, v the advance of the vessel through the water, per revolution, and s be the slip, all in feet, we have $\varphi = v + s$, or $v = \varphi - s$. Ordinarily, s varies from $0.15\ \varphi$ to $0.2\ \varphi$; or from 15 to 20 per cent.

If R be the resistance opposed by the water to the motion of the vessel, which is equal to the thrust on the screw-shaft, and to the reaction of the water upon the screw, and n be the number of revolutions of the screw per minute, the net work of the engines, or the work expended in propelling the vessel, will be $n R v = n R (\varphi - s)$. The work expended in slip, will be $n R s$, and the total work expended on the screw, less that expended in overcoming the friction between the water and the screw blades, will be

$$n R (\varphi - s) + n R s = n R \varphi.$$

The net horse-power of the engines will be

$$H\text{-}P_n = \frac{n R (\varphi - s)}{33000};$$

whence the thrust on the screw-shaft, or the resistance opposed to the motion of the vessel, will be

$$R = \frac{33000\, H\text{-}P_n}{n(\varphi - s)} \text{ pounds. } \ldots\ldots\ldots\ldots (86)$$

Thus, in order to determine the resistance of any vessel, in pounds, we have but to distribute the power, due to a known speed and known number of revolutions per minute, of a screw whose dimensions are known, and to substitute the *net*, or *effective*, power in (86).

The percentage of the power applied to the screw shaft, which is expended in slip is, of course, the same as the percentage of slip.

108. *Dimensions of Screw.*—The diameter of a screw is determined by the mean draught aft, of the vessel, and is usually made about equal to that draught.

The pitch varies from 1.25 to about 1.75 times the diameter; ordinarily being about 1.5 times the diameter.

If the speed of the vessel be V knots per hour, the speed in feet per minute will be 101.43 V; and the pitch of the screw and the revolutions of the engines must be so adjusted that

$$n(\varphi - s) = 101.43\,V;$$

whence,

$$n = \frac{101.43\,V}{\varphi - s}. \qquad (87)$$

If the load draught, aft, be one-half the beam, which is not far from the usual proportion, we may establish an approximate relation between the area of the mid-ship section, and the projected area of the propelling surface of the screw, upon a plane perpendicular to its axis.

If d be the load draught, aft, and b be the beam of the vessel, we may put the mid-ship section at, say, $A = 1.75\,b\,d = 3.5\,d^2$.

Taking the diameter of the screw as equal to d and assuming that the projected area of the screw surfaces is $A' = 0.33$ of the area of the circumscribing circle, such projected area will be $A' = 0.33 \times 0.7854\,d^2$.

Then,

$$\frac{A'}{A} = \frac{0.33 \times 0.7854\,d^2}{3.5\,d^2}$$

$$= 0.074;$$

or,

$$A' = 0.074\,A. \qquad (88)$$

If twin screws, of the same diameter and proportions, be employed, the projected areas of the two screws would be

$$A' = 0.148\,A.$$

In this case the speed would be greater, for a given number of revolutions, because of the greater propelling surface; in consequence of which the slip would be less.

XII. General Design of a Steam-Pumping Plant for a City Water Supply.

109. *Requirements.*—A city, having a population of 50,000, requires a present supply of 3,000,000 gallons in 24 hours. The water is to be taken from a river and pumped, through a force-main about 16,000 feet long, into a reservoir whose water surface is to be 231 feet above the mean level of the source of supply.

In order that the present requirement may be met by pumping only during the day time, the rate at which water will be pumped, during 12 hours of each day, will be 6,000,000 gallons in 24 hours.

Future increase in population, and therefore in the demand for water, as well as the importance of having at all times a reserve engine and pumps, justifies the provision of duplicate engines, having an aggregate pumping capacity of 12,000,000 gallons daily. With such an arrangement, the demands of the city may be supplied at all stages between 3,000,000 and 12,000,000 daily.

110. *The Force Main.*—One million gallons daily is equivalent to

$$\frac{1000000}{86400 \times 7.48} = 1.547 \text{ cubic feet}$$

per second. Therefore, for 6,000,000 and 12,000,000 we have $6 \times 1.547 = 9.282$, and $12 \times 1.547 = 18.564$ *c. f. s.*, respectively.

Assume two diameters of force main; say, 24 inches and 30 inches.

Area of a section of a 24-inch main = 3.1416 sq. ft.
" " " " 30 " " = 4,908 " "

The velocities in these mains, when 6,000,000 and 12,000,000 of gallons are passing, are:

a. 6,000,000 in 24 hours in 24-inch main,

$$v = \frac{9.282}{3.1416} = 2.9 \ f.\ s.$$

b. 12,000,000 in 24 hours in 24-inch main,

$$v = \frac{18.564}{3.1416} = 5.8 \ f.\ s.$$

c. 6,000,000 in 30-inch main,

$$v = \frac{9.282}{4.908} = 1.9 \; f. \, s.$$

d. 12,000,000 in 30-inch main,

$$v = \frac{18.564}{4.908} = 3.8 \; f. \, s.$$

Ordinarily, the velocity in a force-main should not exceed 2 feet per second; but when the full capacity of a pumping plant will not be needed until after the lapse of a period of years, economy dictates that a main should be adopted in which, at the outset, the velocity will be somewhat below the normal limit, and in which, ultimately, that limit may be considerably exceeded.

The difference, in cost, of two mains, the one being 30 inches in diameter, and the other having a diameter such that 12,000,000 gallons would pass with a velocity of 2 feet per second, amounts to fully 50 per cent. of the former.

The saving, due to the employment of the larger pipe, consists in a reduction in the friction-head,—during a part of the period when 12,000,000 may be pumped—amounting to no more than 10 per cent. of total head, including friction, due to the use of the 30-inch main, and representing 10 per cent. of the coal consumption during that period.

As the force-main, once laid, may serve for very many years, even after pumping machinery of largely increased capacity will be required, it may be that the larger size would prove the most economical; but it seems better, in view of the uncertainty as to the rapidity with which the demand for water will increase, to adopt the smaller size, or 30 inches.

Loss of Head Due to Friction.—It will be remembered that Blackwell's formula for friction head, designed to cover all contingencies, as angles, bends, etc., is

$$h = \frac{2.3 \, l v^2}{5280 \, d}; \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

in which l = the length of the main, in feet, v = the velocity in feet per second, d = the diameter of the main, in feet, and h = the friction head, in feet,

When 6,000,000 of gallons are passing through the 30-inch main, (*a*) gives

$$h = \frac{2.3 \times 16000 \times 1.9^2}{5280 \times 2.5} = 10.06 \text{ feet.}$$

When 12,000,000 are passing, since the friction head varies as v^2, we shall have

$$h = 2^2 \times 10.06 = 40.24 \text{ feet.}$$

The actual friction head, when about 6,000,000 of gallons are passing, is found to be a little less than 10 feet.

The force-main of the New Bedford, Mass., Water-works (completed 1869) is 2000 feet long, 16 inches in diameter, and, when tested, passed 5,000,000 of gallons in 24 hours. Under these conditions, (*a*) gives $h = 19.9$ feet.

The following table contains certain results which were obtained during the test.

Date. 1869.	Double Strokes of Engine, per minute.	Gallons Pumped in 24 hours.	Duty* per 100 lbs. of Combustible, foot lbs	Friction head h.
Dec. 9.	13.05	5,594,000	56,504,764	17.74
Dec. 10.	12.74	5,461,000	68,675,220	17.26
Dec. 11.	13.53	5,800,000	72,092,233	21.46
Dec. 14.	10.92	4,861,000	72,372,322	13.84

It will be observed that these values of h are all smaller than that given by (*a*), except the third, which was due to nearly six millions of gallons.

Darcy's Formula for Friction Head.—In this, d is the diameter of the main, in *inches;* all other data being represented as in (*a*). The formula is

$$h = 0.00371\,(d + 1) \times \frac{lv^2}{d^2}; \quad \ldots\ldots\ldots\ldots\ldots (b)$$

and it gives, in the New Bedford case, for five millions of gallons in 24 hours, or for $v = 5.52$ *f. s.*,

$$h = 0.00371\,(16 + 1) \times \frac{2000 \times 5.52^2}{16^2} = 15.014 \text{ feet;}$$

which is a substantial agreement with actual results. In this case, however, there were very few bends in the pipe.

*The increase in duty was due wholly to covering, first *one-sixth*, and then *one-third* of the fire-grate with brick, and thus avoiding the necessity of opening the uptake doors.

As a rule, it will be safer to use (*a*).

The diameter of this main was very small, and the resulting velocity excessively great.

It was, however, deemed more economical, by Hon. W. J. McAlpine, the chief engineer, to put in the small main; because of the then small demand for water, and because of the moderate *length* of the main, which gave, even for the excessive velocity, only a moderate friction head.

How to Observe h.—Observe the pressure in the force main, when the engine is running, as it is indicated by the water pressure gauge; reduce this pressure to feet, and add to the result, the height of the centre of the gauge above the water in the well; from this last result, subtract the elevation of the water surface in the reservoir above the water in the well; the difference will be, approximately, the value of h.

Or, observe the pressures, as indicated by the gauge, both when the pumps are in operation and when they are still; the difference between these pressures will be the friction head, in *pounds*, between the gauge and the reservoir. Reduce this difference to feet, and to the result add 2.3 feet,—for friction between the well and the gauge,—and the result will be the value of h.

At Albany, 1876, Darcy's formula gave results which were too large; while at Troy, 1880, its results were too small. In the case of a 30-inch main conveying six millions of gallons in 24 hours, the Darcy formula gave $h = 7.38$ feet, when the observed value was nearly 10 feet.

Thickness of the Main.—In order to determine the proper thickness of the metal of the force-main, let

$p =$ the pressure in the main, in pounds, per square inch.
$d =$ the diameter of the main, in inches.
$r =$ ultimate resistance of cast iron, in pounds, per square inch; say 18,000 lbs.
$f =$ a safety factor $= 5$.
$c =$ a constant to insure good casting.
$t =$ thickness of metal, in inches.

Then,

$$2\,t\,r = p\,d\,f;$$

whence

$$t = \frac{p\,d\,f}{2\,r} + c$$
$$= 0.00028\,p\,d + 0.25 \quad \ldots\ldots\ldots\ldots (c)$$

In our case, the pressure is due to a head of $231 + 40 = 271$ feet; this represents a pressure of $271 \times 0.434 = 117.18$ pounds.

Let us use $p = 120$ pounds. Then (c) gives, for a 30-inch main,

$$t = 0.00028 \times 120 \times 30 + 0.25$$
$$= 1.008 + 0.25 = 1.258 \text{ inches}$$

The profile of the line of the main indicating that it will be, at no point, more than 215 feet below the reservoir, and as the circumstances are such that there will be no water-ram, we will make the thickness of the main $1\frac{1}{8}$ inches on the low ground, and 1 inch from the reservoir to a check valve to be located at an elevation of about 160 feet above the pumps.

Dupuis' formula for thickness of pipes is as follows:

$$t = 3.4\,n\,(0.0016\,d) + c; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d)$$

in which

$n =$ pressure in atmospheres, of 33 feet each.
$d =$ diameter of pipe, in inches.
$c =$ a constant, varying from $0''.4$ for 6-inch pipe, to $0''.36$ for 36-inch pipe.

If $n = 8$, $d = 30$, and $c = 0.27$, (d) gives

$$t = 3.4 \times 8\,(0.0016 \times 30) + 0.27$$
$$= 1.3056 + 0.27 = 1.5756 \text{ inches.}$$

This formula is evidently intended for street mains which may be subject to shocks, or water-ram, due to the sudden closing of valves.

Mr. Kirkwood's formula, is a modification of that of Dupuis', and is as follows:

$$t = \frac{5\,p\,r}{c - p} + x;$$

or

$$= \frac{2.5\,p\,d}{c - p} + x \text{ inches;} \quad \ldots\ldots\ldots\ldots (e)$$

in which c varies from 5000 to 7500 pounds.

If $p = 120$ pounds, $d = 30$ inches, $c = 5000$ pounds, and $x = 0.25$ inch, (e) gives

$$t = \frac{2.5 \times 120 \times 30}{5000 - 120} + 0.25 = 2.09 \text{ inches.}$$

If $c = 7500$ pounds, $t = 1.68$ inches.

The U. S. Ordnance Manual Formula is

$$t = a + \frac{p\,d}{10000} \text{ inches; } \ldots\ldots\ldots\ldots\ldots (f)$$

in which $a = 0.375$ of an inch, for pipes 12 inches or less, in diameter, and 0.875 of an inch, for pipes 100 inches in diameter.

If $a = 0.625$, $p = 120$ pounds, and $d = 30$ inches, (f) gives

$$t = 0.625 + \frac{120 \times 30}{10000} = 0.985 \text{ inch.}$$

Different authorities use coefficients, varying from 1 to 5, to cover effects of water-ram; but as all large gates close gradually, there should be no water-ram in large pump mains. Distributing pipes, or street mains, being subject to shock, from the closing of hydrants and fire-plugs, should be made thicker; using, perhaps, Mr. Kirkwood's formula, and making $c = 7500$ pounds.

111. *Location of Check-Valves.*—One check-valve should be located near the pump house, and should be provided with a $2\frac{1}{2}$ inch by-pass and valve, through which water may pass back to the boilers, when the engines are idle. A second check-valve and by-pass, is to be located at a point about 160 feet above the pumps, as already indicated.

The check-valves relieve the pumps from pressure, when they are idle, and also aid in controlling the water in the main, in case of breaches in the pipes.

112. *Head-wall and Ventilating Pipes.*—At the point where the main discharges into the reservoir, some special arrangement is necessary, in order to insure the integrity of the reservoir, and to prevent its water being siphoned out in case of a breach in the force-main, or in any of its connections.

In Fig. 66, *m m* is the force-main; at its summit *s*, its bottom is to be higher than high water level *l l*, in the reservoir, so that water cannot find its way along the pipe trench. *W* is a head-wall of rubble masonry, in cement, to be founded on rock, if practicable. *v* is a ventilating pipe about 2 inches in diameter open at the top, to prevent the main from acting as a siphon.

113. *The Boilers.*—In fixing upon the coal consumption, we shall assume that our engines, when built, will give a duty of not less than 60,000,000 foot-pounds for each hundred pounds of coal burned; or, which amounts to the same, 600,000 foot pounds per pound of coal.

Now, one U. S gallon weighs 8.35 pounds, and one million gallons weigh 8,350,000 pounds; which also represents the work, in foot-pounds, required to raise a million gallons one foot high.

It follows, then, that the coal required to raise a million gallons, one foot high, will be

$$\frac{8,350,000}{600,000} = 13.92$$

say, 14 pounds.

One horse-power, maintained during an hour, is equivalent to $33000 \times 60 = 1,980,000$ foot-pounds. This, according to our assumed duty, would require

$$\frac{1,980,000}{600,000} = 3.3 \text{ pounds of coal,}$$

per horse-power per hour; which is equivalent to $\frac{1}{3.3} = 0.303$ of a horse-power per pound of coal.

In determining the coal consumption we use the static head, plus about half the extreme friction-head; say, 250 feet.

Now, in order to raise six millions of gallons 250 feet, we shall require

$$14 \times 6 \times 250 = 21,000 \text{ pounds}$$

of coal; this will represent an hourly consumption of

$$\frac{21000}{24} = 875 \text{ pounds.}$$

If the rate of combustion be 10 pounds per square foot of grate, per hour, the requisite grate area will be $\frac{875}{10} = 87.5$ square feet.

This result shows that we shall require three grates $5.5 \times 5.5 = 30.25$ square feet each; or, $3 \times 30.25 = 90.75$ square feet in the three; the rate of combustion will then be something less than 10 pounds per square foot of grate per hour.

There will then be required *three* boilers, which may be made 5 feet in diameter.

The *heating surface* in each boiler we will fix at $41 \times 30.25 = 1250$ square feet.

The *calorimeter* will be 3.75 square feet, and the *area over the bridge-wall*, 3.33 square feet.

The *thickness of the boiler-shell*, will be, for 75 pounds of steam and iron of 50,000 pounds, (11),

$$t = \frac{5 \times 75 \times 60}{2 \times 0.7 \times 50000} = 0.321 \text{ of an inch.}$$

This is slightly (0.0085) greater than $\frac{5}{16}$ths of an inch; but the excess is so slight that $\frac{5}{16}$ths of an inch will be quite safe.

Without going through the details of the calculations, Art. 34, it will be quite sufficient to say, that the estimated heating surface and calorimeter will be furnished by boilers 5 feet in diameter, 17 feet 4 inches long, and containing 58 — $3\frac{1}{4}$ inch tubes.

The boilers are intended to be set as indicated in Fig. 5., with an underground flue leading to the chimney.

Two 6,000,000 engines will require 6 boilers, with an aggregate calorimeter of $6 \times 3.75 = 22.5$ square feet.

114. *The Chimney.*—If of standard proportions, the chimney will be 60 feet high, above the level of the grates, and will have a flue with an area of 22.5 square feet. In this case, however, our rate of combustion is to be only 10 pounds, or $\frac{5}{6}$ of that which we have designated a standard rate; the flue should then be $\frac{5}{6} \times 22.5 = 18.75$ square feet.

Let it be assumed that the chimney will be 100 feet high; in that case the reduced area of flue will be, (16),

$$18.75\sqrt{\frac{60}{100}} = 14.53 \text{ square feet,}$$

which is the area of a circular flue, 4 feet 4 inches in diameter.

The capacity of this chimney will be too great for a single engine, and the dampers will be brought into requisition.

Possibly a portion of the rear of each grate may have to be covered with fire brick, in order to better control the fires, and the steam, when only a single engine is in operation.

115. *The Engines and Pumps.*—These will be of the Holly, Quadruplex, non-compound type. The speed of steam and pump-pistons will be fixed at 90 feet per minute, or 1.5 feet per second. This moderate velocity is taken, in order that the life

of the machinery may be prolonged, and also in order that, ultimately, or on special occasions, the speed and capacity of a set of pumps may be considerably increased beyond their normal values, by a moderate increase in the speed of the engine.

As there will be four pumps, the normal capacity of each will be 1.5 millions, or $1.5 \times 1.547 = 2.32$ *c. f. s.*; and since the speed of piston is to be 1.5 feet per second, we shall have,

$$\text{Area of pump piston} = \frac{volume}{velocity} = \frac{2.32}{1.5} = 1.547 \text{ square feet,}$$

corresponding to a diameter of about $17\frac{1}{4}$ inches. In order, however, to provide for imperfect filling of the pumps, and to provide a small margin, on the side of safety, the pump pistons will be made $17\frac{3}{4}$ inches, and they will have a stroke of three feet; which will also be the stroke of the steam pistons.

The speed of 90 feet per minute, will evidently require $\frac{90}{2 \times 3} = 15$ revolutions per minute.

The pump-valves will be $1\frac{5}{8}$ inches in diameter, and will have a lift, in order to make the entire opening of the valve effective, of from $\frac{3}{8}$ to $\frac{1}{2}$ an inch. Strictly, the lift of the valve should be equal to one-fourth of its diameter. Thus, if d be the diameter of a circular valve opening, the area of the opening will be $\frac{\pi d^2}{4}$; while the area of the cylindrical section of the discharge under the valve, and between it and its seat, will be $\pi d h$; in which h is the lift of the valve. Equating these values and solving for h, we get

$$\pi d h = \frac{\pi d^2}{4};$$

whence,

$$h = \frac{d}{4}.$$

The diameter of the valve being $1\frac{5}{8}$ inches, the proper lift will then be

$$\frac{1\frac{5}{8}}{4} = \frac{1}{4} \times \frac{13}{8} = \frac{13}{32} = 0.4 \text{ inch.}$$

The number of suction and delivery valves, at each end of the pump, should be such that an expenditure of head equal to 0.5 of a foot, will suffice to pass the water through them in such

volume as to supply the pump with its requirement of 2.32 cubic feet per second.

If there be n valves, each having an opening of a square inches, and if v be the velocity with which the water will pass, under a head of 0.5 of a foot, we shall have

$$n\,a\,v = 2.32 \text{ cubic feet.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

The valve-openings being $1\frac{5}{8}$ inches in diameter, the area of each will be 2.0739 square inches, and the aggregate area, of n openings will be 2.0739 n square inches; and the velocity will be

$$v = \sqrt{2gh} = \sqrt{64.32 \times 0.5} = 5.67 \, f.\, s.$$

Substituting those values in (a) we have

$$\frac{n \times 2.0739 \times 5.67}{144} = 2.32;$$

whence

$$n = \frac{144 \times 2.32}{2.0739 \times 5.67} = 28.6 +$$

The number of valves will be, say, 26; as that number can be better accommodated, and as only a small increase in the velocity will be required in order to pass the required quantity of water. No coefficient of contraction has been used, because the form of the passage will be made such as to render the opening almost wholly effective.

It will be interesting to note, in this connection, the relation, between the aggregate area of valve opening and the area of the pump-plunger or piston.

The area of the pump-piston, $17\frac{3}{4}$ inches in diameter, is 247.45 square inches, while the aggregate area of 26 circular openings, each $1\frac{5}{8}$ inches in diameter, is $26 \times 2.0739 = 53.92$ square inches. The ratio of valve opening to area of pump-piston is, therefore,

$$\frac{53.92}{247.45} = 0.218.$$

In cases where the speed of the pump is larger, the number and aggregate area of the valve openings, as well as the ratio of areas, will, of course, necessarily, be greater, in proportion to the speed.

The head required to discharge the required volume of water may be ascertained, experimentally, by placing one of the valves in the bottom of a cylindrical vessel, of known capacity, filling the vessel with water, and noting the *time* required for the entire discharge of the water through the valve.

In making this experiment, it will be remembered that the volume of water which would be discharged through the valve, under a constant head equal to the height of the vessel, is just twice the volume of the contents of the vessel.

The area of the valve opening, multiplied by the velocity due to the height of the vessel, will therefore be equal to twice the volume of the contained water, divided by the number of seconds observed for its discharge.

It seems scarcely necessary to add, that in order to insure the quiet operation of the pump valves, and the most perfect charging of the pumps, a large number of small valves, of small lift are preferable to a small number of large valves with their necessarily larger lift. This feature was adopted by the Holly Manufacturing Company, at the suggestion of the writer, and is now habitually employed by them, with uniformly excellent results.

The load on each pump-piston will be due to its area and the dynamic head against which it acts. In this case the dynamic head is 231 feet, plus the maximum friction head of 40 feet; making a total of 271 feet; the pressure due to which will be $271 \times 0.434 = 117.61$ pounds; add to this 2.39 pounds, for friction in the induction pipe and in the pumps, and we have a grand total pressure of 120 pounds per square inch.

Now, the area of the pump piston being 247.25 square inches, the *load* will be

$$247.25 \times 120 = 29694 \text{ pounds;}$$

which must be made equal to the mean effective pressure of the steam on the steam piston.

The boiler steam pressure has been fixed at 75 pounds above the atmosphere, per square inch; or $75 + 14.7 = 89.7$ pounds above zero; and if the initial cylinder pressure be taken at 73.3 pounds, and the steam be cut-off at one-third stroke, the mean total pressure, per square inch, per stroke, will be

$$p_m = (73.3 + 14.7\,\frac{(1 + log_e 3)}{3}$$

	$= 88 \times 0.6995 =$		61.56 pounds.
Deduct, for imperfect vacuum..........		2.00	
" " friction....................		2.06	
" " air-pump.................		6.00	
Total deductions....................			10.06 "
Mean net pressure..................			51.50 "

The *area* of the steam-piston must therefore be

$$\frac{29694}{51.5} = 576.6 \text{ square inches;}$$

which corresponds to a diameter of say, 27 inches.

116. *The Fly-Wheel.—The weight of the fly-wheel* will depend upon the excess of the net work of the steam, during a part of the stroke, above the work of the resistance during the same time; or upon the excess of the work of the resistance above the net work of the steam during another part of the stroke, represented by A and B, respectively, in Fig. 57 (in the case of a double engine), and upon the diameter of the path described by the center of gravity of its rim, together with its mean velocity and the permissible variation in its velocity.

Eq. (74), Art. 88, is

$$w = W\frac{(v_1^2 - v_2^2)}{2g};$$

in which w is the work to be stored up in, and given out by, the fly-wheel, in foot-pounds; W, the weight of the fly-wheel, in pounds; v_1 and v_2 the maximum and minimum velocities of the centre of gravity of the rim, in feet per second, and g, the acceleration due to gravity.

Solving for W, we have

$$W = \frac{2gw}{v_1^2 - v_2^2}. \quad \text{.....................} (a)$$

Now, assume that the diameter of the path to be described by the centre of gravity of the rim of the fly-wheel will be 12 feet; its mean velocity, when the engines are making 15 revolutions per minute, will therefore be

$$\frac{15 \times 12 \times 3.1416}{60} = 9.4248 \, f. \, s.$$

Let now the extreme range of speed be supposed to vary from 9 to 10 feet per second; in which case we shall have $v_1 = 10$ and $v_2 = 9$.

Again, referring to the table of Art. 98, it will be observed that the work to be stored in the fly-wheel, in the third case, is 0.0427 of the work of the resistance; and that the work to be given out is 0.0639 of the work of the resistance during corresponding parts of the stroke. We may take a mean of these, or, say, 0.05 of the work of the resistance in the four pumps, during one-fourth of the stroke of one piston, or the work of the resistance in one pump, during an entire stroke of its piston.

We thus get

$$\begin{aligned} w &= 0.05 \times 29694 \times 3 \\ &= 4456.6 \text{ foot pounds.} \end{aligned}$$

Substituting, now, in (a), we get

$$W = \frac{2 \times 32.16 \times 4456.6}{10^2 - 9^2} = 15086 \text{ pounds.}$$

The extreme range of velocity is here one foot, as compared with a mean velocity of 9.5 feet, or rather more than one-tenth of the mean velocity.

If greater regularity be desired, suppose we assume a range only half as great; or from 9.25 to 9.75 feet per second; we shall then have

$$W = \frac{2 \times 32.16 \times 4456.6}{9.75^2 - 9.25^2} = 30172 \text{ pounds.}$$

In a similar case, a fly-wheel having an extreme diameter of 14 feet, and weighing 12 tons, gives perfectly satisfactory results.

We have used the centre of gravity of the *rim* of the wheel, instead of the centre of gyration of the entire wheel, because the regulating effect of the entire wheel is practically the same as it would be if the weight were concentrated in the path described by the centre of gravity of the rim; and also because, where there is more or less of uncertainty in the elements of a calculation, extreme refinements are out of place.

SUMMARY OF RESULTS.

Number of boilers	6.
Length of shell, feet	17.33
Thickness of shell, inches	$\frac{5}{16}$.
Number of tubes in each boiler	58.
Diameter of tubes, inches	3.75
Heating surface, square feet, each boiler	1250.
Grate surface, each boiler, 5.5 × 5.5 =	30.25
" " total, square feet	181.50
Calorimeter, square feet	3.75
Area over each bridge-wall, square feet	3.33
Height of chimney, feet	100.00
Diameter of flue, feet	4.33
Number of pumps, each engine	4.
Diameter of pump-piston, inches	17.75
Stroke of " " feet	3.
Number of valves, each pump, 4 × 26	104.
Diameter of pump-valves, inches	1.625
Lift of " " "	0.4
Number of steam cylinders, each engine	4.
Diameter of steam pistons, inches	27.
Length of stroke, feet	3.

The air-pump, condenser, and feed-pumps, as well as other minor details are omitted, because they are best determined by the draughtsman who designs and arranges the details of the machinery—under the supervision and direction of the engineer,—and because, in these matters, more depends upon the experience and good judgment of the designer than upon refined and elaborate calculations.

117. *Duty Tests of Pumping-Engines.*—The object of a duty test of a pumping engine, is to ascertain the net work, in foot-pounds, resulting from the combustion of 100 pounds of coal. This work is designated the "duty" of the engine.

The test consists in the operation of the engine, under normal conditions, continuously, during a period which, as a rule, ought not be less than 24 hours.

During the test the following records should be kept:

1. The steam pressure, both in the engine and boiler rooms, at intervals of 15 minutes.

2. The vacuum, at intervals of 15 minutes.

3. The barometer, at intervals of an hour.

4. The readings of the counter, at intervals of 15 minutes.

5. The indications of the water-pressure gauge every 15 minutes.

6. The temperature of the air, every 15 minutes.

7. The temperature of the feed-water, every 15 minutes.

8. Tests of quality of the steam, hourly.

9. The quantity of coal used, hourly.

10. The height of the water in the well and in the reservoir, hourly.

11. The weight of the ashes, as drawn.

12. If practicable, or necessary, the quantity of feed water supplied to the boilers.

13. Indicator cards should be taken, from both steam and pump cylinders at frequent intervals.

In starting the test the fires, steam and water should be in their normal conditions; the fires being just ready to receive fresh coal.

During the test all of the conditions should be maintained, as nearly as practicable, and at its conclusion, the water, steam and fires should be in the same condition as at the beginning. The consumption of coal and water will then be represented by the quantities of each supplied to the furnaces and boilers, respectively.

In some cases, steam is raised, with the proper supply of water in the boiler, after which the fires are quickly drawn, and removed, and fresh fires started, with wood and coal; two pounds of wood being counted as equivalent to one pound of coal. At the conclusion of the test the fires are drawn and the unconsumed coal ascertained, as nearly as practicable, and deducted from the quantity supplied during the test, including that used in starting the fresh fires.

With good judgment and proper care, it is believed that the former method leads to equally trustworthy results, besides being simpler and more easily executed.

Care should be taken to see that all instruments are accurate and that all scales and gauges are tested by comparison with proper standards.

The mean net load per square inch on the pump-pistons, will

be the pressure due to the mean static head, plus the pressure due to the mean friction head. The mean static head will be the mean difference in level between the water in the well and that in the reservoir.

The mean friction head will be found as follows:

Take the pressure, per square inch, due to the mean elevation of the water in the reservoir, above the centre of the water pressure gauge in the engine room, from the mean pressure indicated by the water pressure gauge; the difference will be the pressure due to the friction head in the force-main between the gauge and the reservoir; to this add one pound, for the friction of the water in passing through the pump-valves, and such other small pressure as may be deemed sufficient to cover the friction between the well and the pumps.

Reduce the aggregate friction pressure thus found to feet, and add the result to the static head in feet. Thus is obtained finally, the "equivalent lift" of the pumps, in *feet*.

The mean load, in pounds per square inch, will be equal to the static head, reduced to pounds per square inch, plus the pressure due to the friction head; the latter to be determined as already indicated.

If

$p =$ the mean load, per square inch, on the pumps, in pounds,

$A =$ the aggregate area of the pump-pistons, in square inches,

$r =$ the mean number of revolutions, per minute, of the engine,

$s =$ the stroke of the pump-pistons, in feet,

$C =$ the mean rate of coal consumption, in pounds, per minute, and

$D =$ the *duty* of the engine, in foot-pounds, per hundred pounds of coal,

the mean gross load on the pump-pistons will be $A\,p$ pounds; while the space through which this load will be moved, per minute, will be $2\,r\,s$ feet.

Then

$$D = \frac{2 \times 100\,A\,p\,r\,s}{C},$$

$$= \frac{200\,A\,p\,r\,s}{C}, \qquad (89)$$

It is to be regretted that time will not permit the introduction at this point, of an example to illustrate the application of 89, as well as the reduction of the observed data. The student who is specially interested in this direction may, however, supply the omission by procuring a copy or copies of any of the numerous duty tests which have been, and are being made.

XIII. Steam Valves and Valve Motions.

118. *The "D" Slide.*—This valve, which takes its name from its form, is one of the earliest, and is still as largely used, perhaps, as any other. For this reason it is selected, as a representative form, for such illustration and discussion as the brief time at our disposal will permit.

In Fig. 67, V is the valve, covering the steam ports, s, s_1, and the exhaust port E. In this case, it will be observed that the faces, $v\, v$ and $v_1\, v_1$, of the valve, just cover the steam ports. This is the condition of things when the piston P is at the beginning of its stroke. The slightest motion of the valve, to the right, will admit steam to the left-hand end of the cylinder, and will cause the piston to begin its stroke toward the right.

Tracing, now, the connections with the valve-stem f, through $f\, r$, $r\, R$, $R\, r$, and $r_1\, c$, to the main shaft, we observe two eccentric circles; the smaller of these is a section of the main shaft, and the larger is the "eccentric," which is keyed to the shaft and revolves with it; performing, in its revolution, the functions of a *crank*, whose arm is the distance between the centres of the shaft and the eccentric.

This arm, or eccentricity, is termed the "throw" of the eccentric, because that is the distance which the centre of the eccentric is thrown to the right and left of the centre c of the shaft.

This throw of the eccentric, acting through the eccentric-rod $c\, r$, communicates an oscillating motion to the extremity r_1 of the rocker-arm $r\, r_1$, which vibrates about the fixed centre R, and thus communicates a reciprocating motion to the valve-stem f, and through it a reciprocating motion to the valve V. The positions of the valve V, of the piston P, of the rocker-arm $r\, r_1$, of the crank arm $C\, K$, and of the throw $C\, c$ of the eccentric, will next be observed.

The valve V is in its central position, as is the rocker-arm; the shaft, as indicated by the arrow, is revolving with the hands

of a watch. Now, as the crank-pin K passes toward K_1, c will approach the line $K\ K_2$; r_1 will move to the left, and r, f and V to the right.

When c reaches the line $K\ K_2$, the valve will have wholly uncovered the port s, the piston P will be at mid stroke, and the crank-pin will be at K_1.

At the instant that the valve began to open the port s, it will be observed that it also began to uncover the port s_1, and to open communication between the right-hand end of the cylinder and the exhaust passage E, which will be fully opened when s is fully opened; thus allowing the steam, which had made the preceding stroke of the piston, to escape, either into the air or into the condenser.

Now, let the motion of the crank-pin continue from K_1 toward K_2; the throw of the eccentric $C\ c$ will at the same time approach the line $K_1\ K_3$, will draw r_1 to the right, and will throw r, f and the valve V to the left.

When the crank-pin reaches K_2, $C\ c$ will be on the line $C\ K_1$, the rocker-arm and the valve V will be in their mid positions, and the valve will wholly close both ports s and s_1. The piston P will then be at the right-hand end of the cylinder, and will thus have completed a full stroke.

The conditions will now be as they were at the beginning; except that $C\ c$ will now be above the shaft, and will be moving from left to right, and that the piston is now at the right-hand end of the cylinder, in readiness to begin its stroke from right to left.

As the crank-pin passes K_2, c will pass to the right of $C\ K_1$ and will cause the valve to move to the left, from its central position, and thus open the port s_1 and admit steam to the right-hand end of the cylinder.

When the crank-pin arrives at K_3, the steam valve will be at the left of its travel and the port s_1 will be wide open. As the crank-pin moves from K_3 toward K, the valve will begin to close —$C\ c$ now moving toward the left,—and when it reaches K, a complete revolution of the engine will have been completed, and *all* of the original conditions will be restored.

It will, of course, have been observed, that when the valve began to open the port s_1, it also began to open the port s to the exhaust, and thus permitted the steam which had made the stroke from left to right to be exhausted.

It will also have been observed, that the steam ports are open for the admission of steam during the entire stroke of the piston; in other words, that the steam follows full stroke; also, that the valve opened during the first half of the stroke, and closed during the last half of the stroke.

Next, let us observe the relative positions of the crank-arm and the throw, $C\ c$, of the eccentric. It will be seen that the latter is always 90° *behind* the former.

119. *Cut-off by the "D" Slide.*—In order to cut off the steam before the end of the stroke, it is clearly necessary to give the valve such dimensions that the steam ports will be open, for admission of the steam, only during a part of its travel; which, as has already become apparent, is equal to twice the throw of the eccentric. This is done by increasing the length of the valve, at both ends; the added portions projecting beyond the steam-ports, when the valve is in its central position, and constituting what is termed "lap" on the steam side. The dotted extensions at the end of the valve, in Fig. 67, represent the "lap" at each end.

Now, the piston being at the left-hand end of the cylinder, and the throw, $C\ c$, of the eccentric being vertically below the link $K\ K_1$, and moving to the left, it is clear that the steam port, s, will not be uncovered until the point c has moved through a distance equal to the lap, to the left. In the meantime the piston will have completed a part of its stroke to the right. But the steam port must be opened, at the latest, as soon as the piston *begins* its stroke; and, in order to accomplish this, the eccentric must be turned ahead on the shaft, through an arc θ, whose versed sine is equal to the lap. The valve will then be just on the point of uncovering the port when the piston is at the beginning of its stroke; or, technically, when the engine is "on its centre."

The valve will then continue to open, to the right, while the centre, c, of the eccentric travels through the angle $m\ C\ K = 90° - \theta$; as the eccentric, or its centre c, continues its motion about C, to the right, the valve will begin to close the port s, and when it has moved through the angle $K\ C\ n = 90° - \theta$, the port s will be wholly closed and the steam will be cut off. During this period of admission, and while the centre of the eccentric has been moving through the angle $2(90° - \theta) = 180° - 2\theta$, the crank-pin K has moved through the angle $K\ C\ N$, which is

also equal to $180° - 2\theta$. The steam will therefore act expansively, while the crank-pin is passing through the arc NK_2, which measures the angle 2θ at the centre of the shaft, and while the piston is moving through a portion of its stroke which, but for the derangement due to the connecting-rod, would be equal to the versed sine, kK_2, of the arc NK_2.

But in this condition of things, when the steam port s is on the point of being uncovered, owing to the movement of the valve to the right, through a distance equal to the "lap," the steam port s_1 will be opened to the exhaust, by the same amount; in other words, the port s_1 will be opened to the exhaust when the crank-pin is below K, and when it has still to traverse an arc equal to α, before it reaches K, and before the piston will complete its stroke from right to left.

What has been pointed out as occurring while the piston is making a stroke to the right, and while the crank-pin is traversing the arc $K\ K_1\ K_2$, will also occur in connection with the right-hand end of the valve, and the port s_1, while the piston is making its return stroke, from right to left, and while the crank-pin is describing the path $K_2\ K_3\ K$.

It thus appears that, as a result of the addition of "lap" on the steam side of the valve, and in consequence of setting the eccentric forward, which becomes necessary by reason of such lap, the exhaust will be opened *too early*, and that there will be a consequent loss of pressure on the piston, during the latter part of its stroke.

Again, since the exhaust will be opened to one port while the eccentric and crank are making half a revolution, and since this half revolution begins a short time before the piston completes a stroke, it must end, and the exhaust be closed, an equal time before the completion of the following stroke.

Lap, on the steam side alone, thus causes all of the functions of the valve, except its opening for the admission of steam to the cylinder, to be performed too early.

Early opening of the exhaust, as before stated, causes a loss of pressure during the latter part of the stroke; while early closing causes cushioning, or excessive resistance, on the exhaust side, during the latter part of the stroke.

The early opening of the exhaust, and the resulting loss of pressure, are partially remedied by "lap" on the exhaust side, which is indicated by the dotted additions on the inside, and at

each end of the valve; this exhaust lap, however, reduces the time during which the exhaust is open for the escape of the steam; and it does this by causing the exhaust to open *later*, and close *earlier*, than it otherwise would. The earlier closing of the exhaust increases the cushioning. On this account a compromise is necessary, which is effected by making the exhaust lap somewhat *less* than the steam lap.

Thus a portion of the loss of pressure, during the latter part of the stroke is avoided, while cushioning, to a certain extent, is permitted.

While this arrangement is rather desirable, in high speed engines, it obviously imposes a limit upon the measure of expansion which is practicable with the simple "*D*" slide.

This limit is between 1.6 and 1.5, corresponding to a cut-off from $\frac{5}{8}$ to $\frac{2}{3}$ of the stroke.

When higher measures of expansion are desired, the steam passes through rectangular ports in the valve, to the cylinder ports, and an independent cut-off slide-valve is placed on the back of the steam-valve; the independent, or cut-off, valve being worked by a separate eccentric.

120. *The Stephenson Link.*—The valve-motion illustrated in Fig. 67, and described in the preceding article, will permit the engine to run only in one direction. In other words, while the piston has its reciprocating motion, the shaft and crank can revolve only in a single direction—that indicated by the arrow, in Fig. 67.

Locomotive and steam-ship engines must, however, be so arranged as to run equally well in *both* directions; in order that the locomotive or steam-ship may be backed as well as run forward.

The reverse motion is effected by the addition of a second eccentric, which is termed a backing eccentric. The two eccentrics are placed as shown in Fig. 68, in which C is the centre of the shaft, c the centre of the forward eccentric and c' the centre of the backing eccentric. $C\,c$ and $C\,c'$ are the equal eccentricities, or throws, of the forward and backing eccentrics, respectively. Each eccentric is set ahead, for its own motion, on account of "lap," an angle equal to θ. The small dotted circle is a projection of the paths of the centres of the two eccentrics upon a plane perpendicular to the axis of the shaft, and the large dotted circle is the path of the crank-pin. $c\,l$ and $c'\,l'$

are the forward and backing eccentric rods, respectively, and $L L'$ is the link.

The block b is the link block which grasps a pin at the lower end of the vibrating arm $r r_1$, which is identical with the arm $r r_1$ of Fig. 67. The link-block slides in the circular slot, $s s'$ of the link, as the latter is raised or depressed by the lever $H H'$.

As shown in the diagram, the link-block is in the centre of the slot $s s'$; and if the shaft and its eccentrics be made to revolve, the ends of the link, $L L'$, will vibrate in opposite directions, without materially changing the position of the link-block, or that of the arm $r r_1$, and consequently without causing sufficient motion of the valve—now in its central position—to uncover either steam-port. Thus, while both eccentrics may be revolving, neither steam-port is opened, and, although the piston may move, it will be moved by the machinery, and not by the steam.*

To give the cylinder steam, to go ahead, throw the handle H, of the lever $H H'$, to the left, and drop the link so that the link-block will be at s; the forward eccentric, whose centre is c, will now operate the upper end of the link, and through it and the arm $r r_1$, the valve; while the lower end of the link will vibrate, under the influence of the backing eccentric, but without materially effecting the motion of the link-block, or that of the valve.

To back the engine, throw H to the right, and raise the link until the link-block is at s'; then the motion of the steam valve will be reversed, as will that of the engine.

The action of the link may be best studied in connection with the locomotive.

121. *Lead of the Steam Valve.*—Thus far we have supposed the valve to be just on the point of uncovering the steam-port when the engine is on its centres, or when the piston is on the point of beginning its stroke. In the practical adjustment and operation of the steam engine, however, it is customary to have the port slightly uncovered at the beginning of each stroke of the piston; and the distance through which the valve may have

*Usually, the eccentric-rods are not *crossed*, as shown in Fig. 68, but are *open*, the lower end of the link being connected to the eccentric whose centre is at c, and the upper end of the link being connected with the eccentric whose centre is at c'. In such case, however, the ports will nearly always be opened, even when the link is in mid-gear; and when thus opened the engine cannot be stopped by putting the link in mid-gear.

been moved since the beginning of the opening of the port, is termed the "lead" of the valve.

The lead is always small, rarely exceeding an eight of an inch.

In order to produce "lead," therefore, the eccentric must be set forward an angular distance equal to $\theta + \theta'$, instead of θ simply.

The steam valve will then continue to open the port while the crank-pin describes an arc of $90° - (\theta + \theta')$; after which it will begin to close, and, when the crank-pin has moved through an additional arc of $90° - \theta$, will be wholly closed and will cut off the steam admission.

The total arc through which the crank-pin moves, between the beginning of the stroke of the piston and the closing of the steam-port, is, therefore,

$$90° - (\theta + \theta') + 90° - \theta = 180° - 2\theta - \theta';$$

while the arc traversed by the crank-pin, during the period of expansion is $2\theta + \theta'$.

The various forms and modifications of the slide valve, as well as the various means adopted for balancing it, are subjects for special study.

XIV. Analytical Discussion of the Motion of the Slide Valve.

122. *Motions of the Eccentric, Valve and Crank Traced.*—In Fig. 69, a slide valve is shown in all of its principal positions, during a revolution of the engine, together with the corresponding positions of the crank-pin and connecting-rod.

The following is a detailed description of the diagram:

First.—C is the position of the crank-pin, when the piston is on its outer centre; CP is the position of the connecting-rod, and E and VV are the positions of the centre of the eccentric and of the valve, respectively, at the same instant.

Second.—C_1, $P_1\,C_1$, E_1 and $V_1\,V_1$ are the positions of the same elements, respectively, when the steam-port B is wide open.

Third.—C_2, $P_2\,C_2$, E_2 and $V_2\,V_2$ are the positions when the port B is fully closed.

Fourth.—C_3, $P_3\,C_3$, E_3 and $V_3\,V_3$ are the positions at the instant when the piston is on its inner centre.

Fifth.—$C_4 . P_4 C_4 . E_4$ and $V_4\ V_4$ are the positions when the port A is wide open.

Sixth.—$C_5 . P_5 C_5 . E_5$ and $V_5\ V_5$ are the positions at the instant that the port A is fully closed.

Seventh.—$V\ V'$ is the central position of the valve.

Now, let

$$l_2 = V_2\ V_2 = V'\ V_3 = \text{“lap” on the steam side.}$$
$$l_1 = V'\ V_1 = V_3\ V_3 = \text{“lead” on the steam side.}$$
$$t = S\ E_4 = S\ E_2 = \text{“throw” of the eccentric.}$$
$$p = V_4\ V_5 = V_1\ V_2 = \text{width of the steam port.}$$

Then,

$$S k + k\ E_1 = S\ E_4 = S\ E = S\ k' + k'\ E_4 = t;$$

but

$$S\ k' = l_1, \text{ and } k'\ E_1 = k'\ E_4 = p;$$
$$\therefore t = l_2 + p. \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (a).$$

Again, we have

$D\ C =$ distance through which the steam follows, during the inner stroke of the piston.

$D_1\ C_1 =$ distance through which the steam follows, during the outer stroke of the piston—the inner and outer strokes being *toward* the inner and outer ends of the cylinder, respectively;

E' and E' are the positions of the eccentric, corresponding to the central positions of the valve, and $e\ e'$ are the positions of the eccentrics, at the instants when the steam is about to be admitted to the outer and inner ends of the cylinder, through the ports B and A, respectively.

Let

$$\theta = E'\ S\ e = \text{angle due to the “lap” } l.$$
$$\theta' = E'\ S\ E = \text{angle due to the “lap”} + \text{the lead.}$$

Then,

$$\theta' - \theta = e\ S\ E = \text{angle due to the “lead,” } l_1.$$

In order that the valve may have the requisite steam lead, it will be observed that, when the piston is on its outer centre, the eccentric must have moved, from E', its central position, through an angle $E'\ S\ E = \theta'$, whose sine is equal to the lap, plus the lead; hence,

$$E\ S\ sin.\ \theta' = l_1 + l_2;$$

or,

$$t\ sin.\ \theta' = l_1 + l_2$$

$$\therefore sin.\ \theta' = \frac{l_1 + l_2}{t}$$

But, (*a*),

$$t = l_1 + p;$$

which, substituted in the preceding equation, gives

$$sin.\ \theta = \frac{l_1 + l_2}{l_1 + p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(90)$$

Similarly,

$$E\ S\ sin.\ \theta = l_1;$$

or,

$$t\ sin.\ \theta = l_1;$$

whence,

$$sin.\ \theta = \frac{l_1}{t}.$$

Substituting $l_1 + p$ for t, in this equation, we get

$$sin.\ \theta = \frac{l_1}{l_1 + p}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(91)$$

In this case, then, we observe that the radius of the eccentric must make an angle $90° + \theta'$ with the crank, and that it must be set *ahead* of the crank. If, however, the eccentric operates the valve through the intervention of a lever, which has the effect of reversing its motion, the eccentric must be set behind the crank, and at an angular distance of $90° - \theta'$ from it; or, it must be 180° from the first position. Continuing, now, the motion, in the direction indicated by the arrows, we see that the valve will continue to open until the eccentric reaches E_1, when the port B will be wide open. The eccentric will then have advanced through an angle of $90° - \theta'$; and since the crank must have advanced through an equal angle, the position of the crank-pin will now be C_1; making the arc $c\ c_1 = E\ E_1$.

As the motion of the eccentric continues, the valve will begin to close the port B; when the advance $E_1\ n$, beyond E_1 becomes $90° - \theta'$, the port B will be open, to an extent equal to the lead, and when a still further advance, $n\ E_2 = e\ E = E'\ E - E'\ e = \theta' - \theta$, the port B will be wholly closed, and the admission of steam suppressed.

Hence the entire advance of the eccentric, and consequently of the crank, since the crank-pin was at C, has been

$$\begin{aligned} & E\,E_1 + E_1\,n + n\,E_2 \\ & = 90° - \theta' + 90° - \theta' + \theta' - \theta \\ & = 180° - (\theta + \theta'). \quad \ldots\ldots (92) \end{aligned}$$

If θ' be regarded as the advance of the eccentric due to the lead of the valve, (92) may be written $180° - \theta - (\theta + \theta') = 180° - 2\,\theta - \theta'$; which is identical with the advance found in Art. 121.

The angular space, therefore, through which the crank and eccentric must pass, before the completion of the stroke of the piston, is

$$\begin{aligned} C_2\,S\,C_3 &= 180° - 180° - (\theta + \theta') \\ &= \theta + \theta'. \quad \ldots\ldots (93) \end{aligned}$$

Referring to (92), it will be observed that the portion of the stroke of the piston which is completed before the port B is closed, or before cut off, will increase as $\theta + \theta'$ diminishes; it therefore follows that, in order to cut-off shorter, the angles θ and θ' must be increased; in other words, $l_1 + l_2$, must be increased.

But, in any given case, l_2 may be regarded as constant; hence any change in the value of $l_1 + l_2$, must be made wholly in the value of the "lap" l_1.

Generally, l_2 will have a fixed value, and l_1 may vary between the limits zero and the maximum practicable value, which will be considered presently.

If $l_1 = 0$, $\theta = 0$, and (93) becomes $180 - \theta'$; in which θ' is the angle due to the "lead" simply; this shows, since $180° - \theta'$ is the angular movement of the crank during the admission of steam, and since l_2 or θ' must always have a finite value, that $180° - \theta'$ must always be less than $180°$, and therefore that the steam can in no case follow full stroke.

On account, however, of the small value of l_2, and therefore of θ' (when l_1 and θ are zero), $180° - \theta'$ will differ only slightly from $180°$, and the stroke may be very nearly completed before the port is closed.

123. *Lap on the Exhaust Side.*—In Fig. 69, we have represented the valve as having a lap x, on the exhaust side, which is slightly less than on the steam side, or l_1.

If there were no lap on the exhaust side, it is obvious that, at the instant of the opening of either steam port, as B, the ex-

haust through the other port would be open to an extent equal to the lap l_1, on the steam side, plus the lead.

As heretofore explained the effect of this condition of things would be to allow the steam to be exhausted from the cylinder before the completion of the stroke of the piston. To obviate this difficulty, the exhaust lap is provided; but in thus remedying, or guarding against one difficulty, another, of at least equal magnitude, is encountered.

In order that this new difficulty may be properly illustrated, understood and appreciated, let us suppose that the exhaust lap x, is equal to l_1. It is clear that, in that case, the exhaust port will be opened at precisely the same instant that steam is admitted to the opposite end of the cylinder. This is unobjectionable; but the exhaust will be *closed*, at precisely the same instant that the opposite steam port is closed, and the steam cut off; which will cause a volume of vapor to be pent up on the exhaust side of the piston.

This pent up vapor must be compressed, into a volume equal to the clearance, during the completion of the stroke; which involves the expenditure of a greater or less amount of work, depending upon the extent of the steam lap, or upon the value of θ'. There results then, a loss of steam pressure upon, and an increase of resistance to the movement of, the piston during the latter part of its stroke.

In those cases where the vacuum is promptly formed, at the beginning of the stroke, and is well maintained, the increased resistance will not, ordinarily, amount to more than is required for the purpose of forming a proper cushion for the piston at the end of its stroke; but where the vacuum is poor, and especially in the case of non-condensing engines, this resistance assumes a magnitude which may become entirely inadmissible.

In practice, a compromise is effected between the two evils, resulting from the premature opening and closing of the exhaust. This is done by making the exhaust lap x less than the steam lap l_1, in the proportion of about one to two.

Even this, however, does not enable us to use excessive steam lap, for high measures of expansion; a steam lap which causes the admission of steam to the cylinder to be suppressed at about $\frac{5}{8}$ of the stroke, from the beginning, is as great as can be used advantageously, in practice.

Continuing, now, the motion of the eccentric and crank, from the positions in which they were left, Art. 122, until the centre of the eccentric reaches E_3, 180° beyond E, the valve will be in the position V_3 V_3, and the port A will be opened to an extent equal to V_3 V_5, equal to the lead l_2; the crank, having moved through an equal angular distance, or 180°, from its first position, will be at C_3, and the piston will have just completed its stroke.

Continuing the motion still further, until the centre of the eccentric shall have moved through an arc of $90° - \theta'$, and arrived at E_4, the port A will be wide open, and the crank-pin will be at C_4.

The valve will now gradually close the port A, and when the centre of the eccentric shall have moved through an additional arc, $E_4\, m = 90° - \theta'$ the valve will be in the position V_3 V_3, and the port A will be open to an extent equal to $k'\, o' = l_2$.

A further advance of the centre of the eccentric through the arc $m\, E_5 = \theta' - \theta$, will close the port A, cut off the steam, and bring the crank-pin to C_5.

The entire angular motion of the centre of the eccentric, and of the crank-pin, from E_3, and C_3, will have been, as before.

$$\begin{aligned} &90° - \theta' + 90° - \theta' + \theta' - \theta \\ &\quad = 180° - (\theta' + \theta); \ \ldots\ldots\ldots\ldots\ldots\ldots (94) \end{aligned}$$

and there will remain to be described, before the completion of a semi-circumference, or stroke of the piston,

$$\begin{aligned} C_5\, S\, C &= 180° - 180° - (\theta' + \theta) \\ &= \theta' + \theta = C_2\, S\, C_3, \end{aligned}$$

as before.

It follows, therefore, from (93) and (94), that the angular spaces described by the crank, during the admission of steam at the two ports A and B, are equal; and that the spaces described by the crank-pin, during the two periods of expansion are also equal.

124. *Unequal Spaces Described by the Piston, During the Admission of Steam, at the Opposite Ends of the Cylinder:*

Referring, again, to Fig. 69, it will be seen, further, that while the angular spaces described by the crank and eccentric, during the periods of admission of steam at both ends of the cylinder

are *equal*, the spaces described by the piston, in the same times, are *unequal.*

For, disconnecting the connecting-rod at C_2, and revolving it about P_2 as a centre, the end C_2 will cut SC_2 in D; showing that when the crank-pin had reached C_2, and when the steam had been cut off, the piston had traversed a space equal to $C D$, during the period of admission to the outer end of the cylinder. Again, disconnect C_5 and revolve $C_5 P_5$ about P_5; C_5 will cut CS in D_1, and $C_3 D_1$ will be equal to the space through which the piston moved, from the inner end of the cylinder, during the period of admission.

It will be remembered that the arcs $C\ C_1\ C_2$ and $C_3\ C_4\ C_5$, through which the crank-pin passed, during the two periods of expansion, are equal.

$C D$ and $C_3\ D_1$, however, which represent the corresponding movements of the steam piston, are unequal.

Now,

$$C D = C S + S F + F D,$$

and

$$C_3 D_1 = C_3 S + S F_1 - F_1 D_1;$$

but

$$C S = C_3 S,\ S F = S F_1 \text{ and } F D = F_1 D_1;$$

$$\therefore C D - C_3 D_1 = 2F D = 2F_1 D_1. \quad \ldots\ldots\ldots\ldots (95)$$

125. ***Relations of F D to Measure of Expansion, and Length of Connecting-rod.***—In Fig. 69, let

c = the length of the connecting-rod, $P C$.

$\alpha = C_2 P_2 F = C_5 P_5 F_1$ = the angle made by the connecting-rod with the axis of the cylinder, at the instant of cut-off.

Then,

$$\begin{aligned} D F &= P_2 D - P_2 F \\ &= c - c \cos.\ \alpha \\ &= c\,(1 - \cos.\ \alpha) \\ &= c\left(1 - \sqrt{1 - \sin.^2 \alpha}\right) \\ &= c\left(1 - \sqrt{1 - \frac{C_2 F^2}{c^2}}\right). \quad \ldots\ldots\ldots\ldots (a) \end{aligned}$$

Now suppose c to be constant; then $D F$ will increase with $C_2\ F$, and will be a maximum when $C_2\ F$ becomes $H\ S$; or,

when the steam is cut off at the instant the crank completes one-half of the angular movement due to a stroke of the piston, and becomes perpendicular to the axis of the cylinder.

If in (a), $C_2 F$ be constant, $D F$ will diminish as c increases, and if c become infinite, $D F$ will reduce to zero, and the steam will follow equal distances in the two ends of the cylinder.

It is therefore clear, that the longer the connecting-rod, the smaller inequality in the distances traversed by the piston during the periods of admission at opposite ends of the cylinder.

In practice, the connecting-rod ought not to be less than 1.8 to 2, as compared to the stroke of the piston. See Art. 93.

125. *Absolute Advance of the Piston, during the period of Admission.*—Fig. 69. For convenience, let the notation be changed, and let

$c =$ the length of the crank-arm, $S C$.
$n c =$ the length of the connecting-rod.
$a =$ a decimal, expressing the fraction of the stroke from the *outer* end of the cylinder, completed during the period of admission.
$a' =$ a decimal expressing the fraction of the stroke, from the *inner* end of the cylinder, completed during the period of admission.

Then, from the outer end of the cylinder, we have

$$a = \frac{C D}{2c};$$

but $C D = C S + S F + F D$,

$$\therefore a = \frac{C S + S F + F D}{2c}. \quad \ldots\ldots\ldots\ldots\ldots (b)$$

Now, $C S = c$, $S F = c \cos. (\theta' + \theta)$, and

$$F D = P_2 D - P_2 F$$
$$= n c - \sqrt{n^2 c^2 - c^2 \sin.^2 (\theta' + \theta.)}$$

Collecting terms, factoring, and substituting in (b) we get

$$a = \frac{c\left(1 + \cos. (\theta' + \theta) + n - \sqrt{n^2 - \sin.^2 (\theta' + \theta)}\right)}{2c}$$

$$= \tfrac{1}{2} [1 + n + \cos. (\theta' + \theta) - \sqrt{n^2 - \sin.^2 (\theta' + \theta)}]. \quad \ldots (96)$$

We have for the difference between the distance traversed by the piston, during the period of admission, (95)

$$a - a' = \frac{2FD}{2c} = \frac{FD}{c}$$

$$\therefore\ a' = a - \frac{FD}{c}$$

$$= a - \tfrac{1}{2}\left(n - \sqrt{n^2 - \sin.^2(\theta' + \theta)}\right)$$

$$= \tfrac{1}{2}[1 - n + \cos.(\theta' + \theta) + \sqrt{n^2 - \sin.^2(\theta' + \theta)}]; \quad \ldots\ldots(97)$$

also.

$$a - a' = n - \sqrt{n^2 - \sin.^2(\theta' + \theta)}. \quad \ldots\ldots\ldots\ldots\ldots(98)$$

If, $(\theta' + \theta) = 0$, we have, from (98),

$$a - a' = n - \sqrt{n^2} = 0;$$

which corresponds to the case where steam follows full stroke.

If, $(\theta' + \theta) = 90°$, which renders $a - a'$ a maximum, (98) becomes

$$a - a' = n - \sqrt{n^2 - \sin.^2 90°}$$

$$= n - \sqrt{n^2 - 1}.$$

127. *To Find $(\theta' + \theta)$ for any Value of a.*—Clearing (97) of fractions, and transposing, we get

$$2a - 1 - n - \cos.(\theta' + \theta) = -\sqrt{n^2 - \sin.^2(\theta' - \theta)};$$

whence

$$\cos.(\theta' + \theta) = \frac{4an + 4a - 4a^2 - 2n - 2}{2(1 + n - 2a)},$$

$$= \frac{2an + 2a - 2a^2 - n - 1}{1 + n - 2a},$$

$$= \frac{2a(n + 1 - a) - n - 1}{1 + n - 2a}. \quad \ldots\ldots\ldots(100)$$

If, in (100), the value of $\cos.(\theta' + \theta)$, in terms of l_1, l_2 and p, be substituted, the resulting equation may be solved for l_1, and the steam lap be thus accurately determined; but the process would be complicated and tedious.

We may, however, determine l_1 with sufficient accuracy for practical purposes, in a less laborious manner, as follows:

If, in (90) and (91), we consider $\theta' = \theta$, and make $\frac{1}{2}(\theta' + \theta) = \theta_1$, we shall get, (91),

$$l_1 = l_1 \, sin. \, \theta_1 + p \, sin. \, \theta_1;$$

or,

$$l_1 (1 - sin. \, \theta_1) = p \, sin. \, \theta_1;$$

$$\therefore l_1 = \frac{p \, sin. \, \theta_1}{1 - sin. \, \theta_1}$$

$$= \frac{p}{cosec. \, \theta_1 - 1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (101)$$

If the valve be without "lead," $\theta' = \theta$ and (101) will give correct results.

Let it be assumed, now, that $\theta' - \theta = 2°$; then (101) becomes

$$l_1' = \frac{p}{cosec. \, (\theta_1 - 1°) - 1}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

and since the approximate "throw" will be equal to p + the approximate lap l_1',

$$t' = p + l_1'$$

$$= p + \frac{p}{cosec. \, (\theta_1 - 1°) - 1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

Let l_2' be the "lead" due to this assumed difference, of 2°, between θ' and θ; then

$$l_2' = t' \, sin. \, (\theta_1 + 1°) - l_1'. \quad \ldots\ldots\ldots\ldots\ldots (c)$$

Substituting, now, for t' and l_1' in (c), their values in (a) and (b), we get

$$l_2' = \frac{p \, cosec. \, (\theta_1 - 1°) \, sin. \, (\theta_1 + 1°)}{cosec. \, (\theta_1 - 1°) - 1} - \frac{p}{cosec. \, (\theta_1 - 1°) - 1}$$

$$= \frac{p \left[cosec. \, (\theta_1 - 1°) \, sin. \, (\theta_1 + 1°) - 1 \right]}{cosec. \, (\theta_1 - 1°) - 1}. \quad \ldots\ldots\ldots (102)$$

The value of l_2' has been found by making the angle θ, due to the lap, 2° less than the angle θ' due to the lap plus the lead; or 1° less than $\frac{1}{2}(\theta' + \theta)$; which is $\theta_1 - 1°$.

Now, since $\theta' - \theta$ will always be a small angle, we may, without material error, assume that the leads due to diminished values of θ_1 will be proportional to such diminution; in other words, that the leads due to $\theta_1 - 1°$, $\theta_1 - 2°$, $\theta_1 - 3°$, etc., will be proportional to 1, 2, 3, etc.

Let, then, δ = the angle which must be substracted from θ_1 in order to find θ, or

$$\delta = \theta_1 - \theta;$$

then we have

$$l_2' : l_2 :: 1^\circ : \delta;$$

whence

$$\delta = \frac{l_2}{l_2'}. \quad \dots\dots\dots\dots\dots\dots (103)$$

If, $l_2 = 0''.15$ and $l_2' = 0''.05$, $\delta = 3^\circ$.

Having found δ, make $\theta' = \theta_1 + \delta$, and $\theta = \theta_1 - \delta$; then find l_1 by (101); putting θ in the place of θ_1.

Then, (*a*) Art. 122,

$$t = l_1 + p.$$

Next, calculate l_2 by (102); using θ' in place of $\theta_1 + 1^\circ$, and θ in the place of $\theta_1 - 1^\circ$.

The result will be sufficiently near the truth for practical purposes, and the error will be indicated by a comparison of the assumed value of l_2, with the result finally obtained from (102).

Example.—Let $a = 0.6$, $n = 3$, $p = 4''$ and $l_2 = 0''.5$; p *and* l_2 being exaggerated for obvious reasons.

Then, (100) gives

$$\cos.\,(\theta' + \theta) = \frac{2 \times 0.6\,(3 + 1 - 0.6) - 3 - 1}{1 - 3 - 2 \times 0.6} = 0.02875$$

$$\therefore \theta' + \theta = 88^\circ\ 21'\ 45'';$$

$$\theta_1 = \frac{\theta' + \theta}{2} = 44^\circ\ 10'\ 52''.5;$$

$$\theta_1 + 1^\circ = 45^\circ\ 10'\ 52''.5;$$

and

$$\theta_1 - 1^\circ = 43^\circ\ 10'\ 52''.5.$$

From (102), we get

$$l_2' = \frac{4\,(\mathit{cosec.}\ 43^\circ\ 10'\ 52''.5 \times \mathit{sin.}\ 45^\circ\ 10'\ 52''.5)}{\mathit{cosec.}\ 43^\circ\ 10'\ 52''.5 - 1.}. = 0.313 \text{ inch},$$

and from (103),

$$\delta = \frac{0.5}{0.313} = 1^\circ\ 36'.$$

We have then,

$$\theta' = \theta_1 + \delta = 44° \ 10' \ 52''.5 + 1° \ 36'$$
$$= 45° \ 46' \ 52''.5$$

and

$$\theta = \theta_1 - \delta = 44° \ 10' \ 52''.5 - 1° \ 36'$$
$$= 42° \ 34' \ 52''.5.$$

We now get, (101),

$$l_1 = \frac{4}{cosec. \ 42° \ 34' \ 52''.5 - 1} = 8''.37.$$

$$\therefore t = l_1 + p$$
$$= 8.37 + 4 = 12.37 \text{ inches.}$$

In order to prove that our method gives results which are practically correct, we have only to substitute, in (102), for $\theta_1 - 1°$ and $\theta_1 + 1°$, their values, as found above. We thus get,

$$l_2 = \frac{4 \ (cosec. \ 42° \ 34' \ 53'' \times sin. \ 45° \ 46' \ 53'' - 1)}{cosec. \ 42° \ 34' \ 53'' - 1} = 0.502 \text{ inch;}$$

which differs only 0.002 of an inch from the assumed lead; a variation which is within the limits of unavoidable mechanical error.

The point of cut-off, here considered, is that which occurs while the piston is moving from the outer to the inner end of the cylinder, or *toward* the shaft, in a direct acting engine; and from the inner to the outer end of the cylinder, or *from* the shaft, in a back-acting engine.

The difference between the distances through which the steam follows, in the opposite ends of the cylinder, is given by (98), and is as follows:

$$a - a' = 3 - \sqrt{3^2 - sin.^2 \ 88° \ 21' \ 45''} = 0.16.$$

$$\therefore a' = a - 0.16 = 0.6 - 0.16 = 0.44.$$

The steam will therefore be cut off at 0.6 of the stroke on one end, and at 0.44 of the stroke on the other; and the *mean* cut-off will be at 0.52 of the stroke.

Example 2.—In the immense engines which were built by the U. S. Navy Department, 1864–1868, for a class of fast cruisers, we have $p = 2.25$ inches, $l_1 = 3$ inches, $l_2 = 0.2$ of an inch (assumed), $c = 24$ inches, and $n = 5$.

Then, (a), Art. 122,

$$t = p + l_1$$
$$= 2.25 + 3 = 5.25 \text{ inches.}$$

Also, (90),

$$sin.\ \theta' = \frac{3 + 0.2}{3 + 2.25} = 0.60952;$$
$$\therefore \theta' = 37° 30'.$$

Again, (91),

$$sin.\ \theta = \frac{3}{3 + 2.25} = 0.57143;$$
$$\therefore \theta = 34° 50'.$$

Hence,

$$\theta' + \theta = 37° 30' + 34° 50'$$
$$= 72° 20'.$$

Then, (96),

$$a = \tfrac{1}{2}(1 + 5 + 0.3035 - \sqrt{5^2 - 0.951^2})$$
$$= \tfrac{1}{2}(6.3035 - 4.908) = 0.7017$$

of the stroke; and, (98),

$$a - a' = 5 - \sqrt{5^2 - 0.951^2}$$
$$= 5 - 4.908 = 0.092$$
$$\therefore a' = a - 0.092 = 0.7017 - 0.092 = 0.6097$$

of the stroke.

The mean cut-off was, therefore,

$$\frac{0.7017 + 0.6097}{2} = 0.6557$$

of the stroke.

The engines were designed to cut off at $\frac{2}{3}$, or 0.667 of the stroke.

Example 3.—Suppose, in example 2, that p, c and n are known, a and l_2 assumed; and let it be required to find l_1 and t. In that case, $p = 2.25$ inches, $c = 24$ inches, and $n = 5$. Assume a and l_2 to be 0.7 of the stroke and 0.2 of an inch, respectively.

Then, (100),

$$\cos.\ (\theta' + \theta) = \frac{2 \times 0.7 \times 5 + 2 \times 0.7 - 2 \times 0.7^2 - 5 - 1}{1 + 5 - 2 \times 0.7} = 0.3087;$$

therefore

$$(\theta' + \theta) = 72^\circ\ 02', \text{ and } \theta_1 = \frac{\theta' + \theta}{2} = 36^\circ\ 01';$$

hence

$$\theta_1 + 1^\circ = 37^\circ\ 01' \text{ and } \theta_1 - 1^\circ = 35^\circ\ 01'.$$

(102) gives

$$l_2' = \frac{2.25\,(cosec.\ 35^\circ\ 01') \times sin.\ 37^\circ\ 01' - 1}{cosec.\ 35^\circ\ 01' - 1}$$

$$= \frac{2.25\,(1.742 \times 0.602 - 1)}{1.742 - 1}$$

$$= \frac{2.25 \times 0.048684}{0.742} = 0.148 \text{ inch.}$$

Then, (103),

$$\delta = \frac{0.2}{0.148} = 1^\circ.35 = 1^\circ\ 21';$$

hence

$$\theta' = \theta_1 + \delta = 36^\circ\ 01' + 1^\circ\ 21' = 37^\circ\ 22',$$

and

$$\theta = \theta_1 - \delta = 36^\circ\ 01' - 1^\circ\ 21' = 34^\circ\ 40'.$$

We get, from (101),

$$l_1 = \frac{2.25}{cosec.\ 34^\circ\ 40' - 1} = \frac{2.25}{1.758 - 1} = 2.968$$

inches; say, 3 inches.

Finally, (*a*), Art. 122, gives

$$t = 2.25 + 2.968 = 5.218 \text{ inches;}$$

say, 5.25 inches.

The slight discrepancies between these results and those in *Example* 2, are due to the fact that in the table of sines used in this example, the angular intervals were quarter degrees, instead of single minutes.

In this case, if there is no exhaust lap, the exhaust will evi-

dently open when the crank-pin is θ degrees from its centres C and C_1, Fig. 69; as that represents the angular movement of the eccentric from E' at which the valve is in its central position.

The movement of the piston, while the crank-pin is describing the arc θ, will be $1 - 0.7 = 0.3$ of the stroke, or $0.3 \times 48 = 14.4$ inches; during which movement it will be cushioning steam on the exhaust side and losing pressure on the steam side; since one steam port will be opened to the exhaust at the instant the other is closed.

If the clearance be equivalent to 10 per cent. of the stroke, or 4.8 inches, the volume of vapor, confined at the close of the exhaust, will be equivalent to $0.3 \times 48 + 4.8 = 19.2$ inches in length, by the area of the piston; and if the tension of the uncondensed vapor be 4 inches of mercury, or 2 pounds per square inch, the tension, after compression, will be

$$\frac{2 \times 19.2}{4.8} = 8 \text{ pounds per square inch.}$$

If, however, the clearance be only 5 per cent., the volume of confined vapor will be $14.4 + 2.4 = 16.8$ inches in length, by the area of the piston, and its tension, at the end of the stroke, will be

$$\frac{2 \times 16.8}{2.4} = 14 \text{ pounds per square inch.}$$

These cushioning effects are not, in themselves, objectionable; but the early exhaust, and consequent loss of pressure on the steam side of the piston *is* objectionable; and enough exhaust lap should be provided to delay the opening of the exhaust, as long as may be practicable, without causing an excess of cushion on the opposite side of the piston, from the early closing of the port A, to the exhaust.

Assume, for example, an exhaust lap of 1.5 inches, or one-half the steam lap.

Then,

$$sin.\ \theta_e = \frac{l_e}{t} = \frac{1.5}{5.25} = 0.2857. \quad \ldots\ldots\ldots\ldots (104)$$

and

$$\theta_e = 16^\circ\ 36';$$

in which l_e is the exhaust lap and θ_e the angular motion of the centre of the eccentric, from its central position at E', due to l_e.

Now suppose φ to be the angular movements of the crank and eccentric during the interval between the opening of the port B to the exhaust, and the opening of the port A, by a distance equal to l_2.

Then, when the crank, approaching the position SC_3, is at an angular distance of φ from it, the centre of the eccentric is an equal distance back of the point E_3; this follows from the fact the crank and eccentric traverse equal angular spaces in equal times; and because, when the crank-pin reaches C_3 the centre of the eccentric will have reached E_3.

At this instant the port B is about to be opened to the exhaust; the valve having moved to the right through a distance equal to $l_1 + l_e$, while the crank has been moving through the angular distance $\theta' + \theta - \varphi$.

But the angular motion of the crank, beyond $C_3 S$, and the equivalent angular motion of the eccentric, beyond $E_3 S$, corresponds to the steam lap l_1, of the valve, and places the valve in its central position.

The eccentric angle θ_e, beyond E', corresponding to the exhaust lap, is therefore found thus:

$$\theta + \theta_e = \theta' + \theta - \varphi;$$

or

$$\theta_e = \theta' - \varphi;$$

whence

$$\varphi = \theta' - \theta_e. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (105)$$

Substituting, now, for θ' and θ_e, their values already found, we get

$$\varphi = 37^\circ\ 22' - 16^\circ\ 36' = 20^\circ\ 46';$$

which, substituted, successively, for $\theta' + \theta$, in (96) and (97), gives, for the fractions of the inward and outward strokes which are completed at the instants when the respective ports are opened to the exhaust,

$$a_r = 0.9738, \text{ and } a_r' = 0.9612.$$

The pistons will therefore be

$$48 - 0.9738 \times 48 = 1.3576 \text{ inches.}$$

and

$$48 - 0.9612 \times 48 = 1.8624 \text{ inches},$$

from the ends of their respective strokes, when the steam begins to escape, or when the exhaust opening begins.

Now it is evident that, when this condition exists, or rather when the port begins to open to the exhaust, the opposite exhaust has been closed for some time, and that, during such time, the valve has moved a distance, to the right, l_e, to its central position, and a further distance, l_e, to the position at which the port B is opened to the exhaust. The total movement of the valve has therefore been $2\,l_e$, and the corresponding angular movement of the eccentric, $2\,\theta_e$; or $2 \times 16° 36' = 33° 12'$.

If φ_c be the angle between $S\,C_3$ and the crank, at the instant when the port A closes to the exhaust, and when the cushioning in the left-hand end of the cylinder begins, we shall have

$$\varphi_c = \varphi + 2\,\theta_e$$
$$= 20° 46' + 2 \times 16° 36' = 53° 48'.$$

This value of φ_c, substituted successively in (96) and (97) determines the fractions of the inward and outward strokes, which are completed at the instants when the ports A and B are closed to the exhaust, or when the cushioning begins in the left and right-hand ends of the cylinder, respectively. Putting a_c and a_c' for these distances to the points where cushioning begins, we get, (96) and (97),

$$a_c = 0.7781 \text{ and } a_c' = 0.7625.$$

The fractions of the stroke during which compression takes place, are therefore,

$$1 - 0.7781 = 0.2219, \text{ and } 1 - 0.7625 = 0.2375;$$

while the actual movements of the piston, during the periods of compression, are $48 \times 0.2219 = 10.65$ inches, and $48 \times 0.2375 = 11.4$ inches, respectively.

Collecting the results found, we have the following tabular statement of the action of the valve:

Stroke.	Fractions of Strokes Completed During			
	Admission.	Expansion.	Release.	Compression.
Inward	0.7017	0.2983	0.0262	0.2219
Outward.............	0.6097	0.3903	0.0388	0.2375

This arrangement does not appear to be objectionable; indeed more exhaust lap and more compression would, in some cases, be desirable.

It is not to be understood that a satisfactory scheme for a valve and link-motion can be designed mathematically; as, in practice, peculiar circumstances of location, imperfections in mechanical arrangements, and elasticity of the materials will impose conditions which can be satisfied only by careful graphical construction, in connection with the theoretical determinations.*

It should be remarked that the unequal periods of admission due to the inclination of the connecting-rod, may be equalized by a proper suspension of the link; and that variable periods of admission are secured by raising or depressing the link so that the link-block b, will occupy positions at variable distances from that which it occupies when in mid-gear.

128. *Automatic Regulation of Engine Speed.*—Modern stationary engines, especially such as are designed to run under loads varying within wide limits, are provided with eccentrics which admit of automatic adjustment to the varying work required of the engine. When running light, just sufficient steam is admitted to maintain a speed only slightly in excess of the normal speed. As the engine is loaded, a slight reduction in its speed causes the eccentric to be so shifted as to admit the requisite quantity of steam to perform the work and to maintain the normal speed.

Fig. 70. shows the arrangement of the Ide "Automatic Safety Governor," divested of some of its details, which is best described in the language of Messrs. Ide & Son, in their circular:

"The Automatic Safety Governor is shown in position in the fly-wheel. The eccentric has a pocket, projecting from the lower side, to which a steel arm (a a') is bolted.

This arm is pivoted near the rim of the wheel. Opposite to this arm is a lug, to which two steel arms (b b) are connected. These arms are connected to two bent steel levers (l l_1 and l' l_1'), on which two fly-weights (W W) are placed.

As these arms move out, into the positions shown by the dotted lines, the eccentric is carried across the shaft.

The movement of the eccentric is obtained from the centrifugal force of the weights, which movement is controlled by the

*The student is referred to an excellent work, by Wm. S. Auchincloss, C. E., entitled, "*Valve and Link Motions*," published by D. Van Nostrand, New York, 1869.

spiral springs (*s s*). The speed of the engine may be changed, to suit requirements, or by tightening or slackening the springs.

"The springs hold the eccentric at a point which gives the valve full throw, until the engine has attained its speed, when the centrifugal force of the weights is greater than the tension upon the springs, when the weights fly out, moving the eccentric into such a position that it shortens the throw of the valve, and at the same time advances the lead.

"The dash-pot (not shown), attached to the end of one of the bent levers, controls the movements of the weights, preventing any sudden movement, or jumping of the weights, when a great change in the load occurs suddenly; and by its use a more sensitive adjustment, of the springs and weights, can be made, and a closer and more perfect regulation of speed can be obtained; it also holds the eccentric in a comparatively rigid position, to overcome the rapidly alternating resistance to the movement of the valve.

"When the engine is running without a load, the valve opens to admit steam, at full boiler pressure, exactly on the centre, and closes before the piston moves one-half inch. Steam is allowed to follow, at full boiler pressure, further and further, as the load is increased.

"When the maximum load is reached, the full throw of the valve is given; and in order to keep up the speed, steam is admitted, at full boiler pressure, up to three-fourths of the stroke.

"The large ports allow a very large opening to be obtained with a small travel of the valve, giving full boiler pressure, in the cylinder, even at a high speed, and when the steam is following over three-fourths of the stroke. The valve will close and cut off steam at a point that will just do the work and maintain the regular speed."

The general principle involved in this automatic cut-off regulator, is that which is employed in most of the best modern stationary engines. The efficiency of these automatic regulators is simply marvelous; the extreme variation in speed, resulting from quickly throwing off the entire load, being in some cases only about *one per cent.* of the normal speed. In a recent test of one of these engines, running at a speed of 275 revolutions per minute, the power was increased from 18 to 40 horse-power, within *five seconds*, with a reduction in speed of less than *one-half of one per cent.*

129. *Double Puppet Valves of Beam Engines.*—The valve used in connection with the beam engines of side-wheel river, lake and ocean steamers, is the double-"puppet," popularly designated the double-"*poppet*," valve.

Fig. 71 shows the form and arrangement of the valves, and the mode of working them.

s s' is the double steam valve, and s_1 s_1' the double exhaust valve, at one end of the cylinder; the upper disc of the steam valve s, is made slightly larger than the lower one, s', in order that an excess of steam pressure, downward on s, over the upward pressure on s', may serve to hold the valve in its seat; the lower disc of the exhaust valve is made larger than the upper one, so that the excess of the downward pressure of the exhaust, on the top of s_1' over the upward pressure on the bottom of s_1, may serve to hold the valve in its seat.

In Fig. 71, only the arrangement of the valves and connections at the top of the cylinder is shown; there is an entirely similar arrangement at the bottom of the cylinder, and the upper and lower steam chests are connected by side pipes, one of which is the steam pipe, and the other the exhaust pipe, leading to the condenser. These pipes are not shown in the diagram.

The valve-stems project upwards, through stuffing-boxes in the tops of the steam chests, and at their upper extremities are clasped by arms, extending horizontally, from vertical lifting rods, placed in front of the cylinder. There are four of these lifting rods; two for lifting the steam valves at the upper and lower ends of the cylinder, and two for lifting the exhaust valves.

C is the centre of the rock-shaft, and $C c$ is the rocker-arm, carrying a pin at c, which is clasped by the hook on the end of the eccentric rod, and which receives a vibratory motion, about C as a centre, from the eccentric.

The main shaft carries two eccentrics; the one operating the steam valves, and the other the exhaust valves. The eccentric-rods lie on opposite sides of the cylinder, and each operates its own rock-shaft.

$C t_1$ are curved arms, attached to the rock-shaft, immediately over which, and attached to the steam lifting rods, are horizontal arms, $l t_1'$ which are forced upward by the action of the curved arms, $C t_1$. Back of, and in line with these parts, are shorter and nearly straight arms, attached to the exhaust rock-shaft, over

which are arms, of corresponding length, attached to the exhaust valve lifting rods, which are raised by the action of the rock-shaft arms, and which are forced down again either by gravity, or by the action of springs.

The exhaust arms and rods are so arranged that the exhaust valves are raised at the beginning of a stroke and closed at the end of the same stroke; each exhaust valve remaining in its seat while the other is opening and closing.

The long curved arms which are attached to the steam rock-shaft, and which operate the steam valves, are so placed that each will lift its valve at the proper instant, and will permit it to close again, before the completion of the stroke which results from the admission of steam which it permits.

In this way the steam is cut off, after a portion of the stroke is completed, and expands during the remainder of the stroke. In order to cut off shorter, the long curved arms are dropped on the rock-shaft, and the steam eccentric is set ahead—in order to open the valve at the proper time.

To follow further, the curved arms are raised, and the steam eccentric is set backward on the shaft—so as to prevent the premature lifting of the valve.

The arrangement, just described, is known as Stevens' cut-off; as originally constructed, it could be adjusted only when the engine was at rest.

Sickles' cut-off differs from that of Stevens, in that the steam valve, while rising, can be tripped, and dropped, at any point; dash-pots being provided to prevent the valves seating themselves with a destructive blow. This cut-off, which is practically instantaneous, is adjustable at the will of the engineer, to cut off at any point of the stroke of the piston.

Winter's cut-off is shown in Fig. 72. *A* is the rock-shaft, which is made to revolve by the eccentric. *B* is a cam on the rock-shaft which revolves with the shaft, and raises the adjustable toe *C*, which in turn raises the lifter *D*, and thus opens the valve. When the point of the cam, as it revolves, passes the point *b*, of the adjustable toe, the latter, and the valve lifting-rod drop, and the valve is closed.

Thus, by moving the adjustable toe *C*, to the right or left, by means of the screw operated by the handle *h*, the period of admission and the point of cut-off are varied, at will.

Examples of cut-offs might be multiplied; but as our pur-

pose is merely to call attention to the thing to be done, and to point out, in a general way, the manner in which, and the means by which, the thing has been or may be accomplished, rather than to enter upon an exhaustive treatment of the subject of valves and their motions, the student is referred, for further information, to the numerous practical examples, in connection with the best modern practice, which are accessible in all parts of the country.

Printed descriptions, illustrated by diagrams, are chiefly valuable, because they show the arrangement and movement of parts which are hidden from view; these may be found in Auchincloss, on "Valve and Link Motions," already referred to, and from the numerous works on the steam engine, by other authors. These should be carefully studied by the learner, who should subsequently direct his attention, as opportunity offers, to the study of the valves and their motions in actual operation.

In the design and general arrangement of valve motions, graphical methods are essential.

APPENDIX.

1. *Heat of Combustion of Coal.*—Kestner and Dollfus have concluded from their carefully conducted experiments, about 1870, that the heat of the combustion of coal cannot be calculated, even approximately from its chemical composition. They say that Dulong's formula, which makes allowance for the H, in the form of water, in the coal, gives results too low; while if the H is credited with its heat, the results are sometimes too low, and at other times too high to agree with experimental results.

Coals experimented upon varied from 96.66 to 76.87 of C, and from 5.1 to 1.35 of H; while the O, H and S varied from 18.45 to 1.99 %.

The lowest experimental result was 8259 centigrade heat-units (14856 Fahr. units); pure C being 8080 centigrade (or 14544 Fahr.) units. This was obtained from the coal having the largest percentage of C, and the smallest of H.

The highest result was obtained from a coal in which $C = 89.96$ and $H = 4.41$. Here there were 9623 cent. units, or 17321 Fahr. units, per pound. Only 9193 cent. units were obtained from a coal in which $C = 89.96$ and $H = 5.09$; which, from its composition, ought to have given more heat than the preceding sample. It would thus appear that we are not able to judge, accurately, as to the relative efficiencies of different kinds of coal from their chemical compositions.

2. *Petroleum as a Fuel.*—Chemically, and experimentally, one pound of oil is found to be equivalent to 1¾ pounds of coal.

When the saving in the cost of labor, due to the use of oil, is considered, it is held that one pound of oil is equivalent to *two* pounds of coal. A gallon of oil weighs 7.3 pounds and is therefore equivalent to 14.6 lbs. of coal.

A long ton of coal is thus equivalent to $\frac{2240}{14.6} = 153.42$ gallons of oil. The question of economy will depend upon the relative costs of the two fuels. The following table shows the costs of oil per gallon and per barrel, and of coal, per ton of 2240 pounds, at which the two fuels are of equivalent value for heating purposes.

Oil at		Is Equivalent to Coal at
Per Gallon.	Per Barrel.	Per Ton.
Cents.	\$ c.	\$ c.
1	0.42	1.53
1½	0.63	2.30
2	0.84	3.07
2½	1.05	3.83
3	1.26	4.60
3½	1.47	5.37
4	1.68	6.14
4½	1.89	6.90
5	2.10	7.67

Where oil is cheap and coal dear, the former may be used as a fuel, economically.

At Ann Arbor, Mich., a recent trial has shown that the substitution of oil for coal reduces the cost of a given amount of work from \$10.50 to \$6.50. The prices of oil and coal are not stated.

Mr. Thomas Urquhart, Locomotive Superintendent of the Grazi Tsaritzin Railway, in south-eastern Russia, where oil is abundant and cheap, and where other fuel is scarce and dear, has 143 locomotives in constant use burning petroleum.

A jet of steam is thrown, horizontally, into the furnace. Surrounding the steam jet is a stream of oil and surrounding this is an annular opening for the admission of air. The rates of admission, of the steam, oil and air are regulated to meet the requirements of each case. The oil is converted into a spray by the rapidly expanding steam and mingles with the air which supports the combustion.

3. *Water Gas as a Fuel.*—Mr. William Kent, Mechanical Engineer, in a paper published in the *Railroad and Engineering Journal*, January, 1887, treats this subject, substantially as follows:

$$\text{Water gas} = CO + 2H.$$

When burned, the products of combustion consist of

$$CO + 2H + 2O + 2N + \text{excess of air.}$$
$$= CO_2 + H_2O + 2N \quad + \quad \text{“} \quad \text{“} \quad \text{“}$$

$CO + 2H$, from $C + H_2O$, weighs

$$12 + 16 + 2 = 12 + 18;$$

and when burned, the products are

$$CO_2 + H_2O + 2N + \text{excess of air.}$$
$$= 12 + 32 + 2 + 16 = 44 + 18,$$

from the combustion of 12 pounds of C. The number of heat-units in 12 lbs. C. is

$$12 \times 14500 = 174000;$$

while the heat due to 18 H_2O (above 78°) is

$$18 \times 1100 = 19800 \text{ units;}$$

which represents the loss of heat in the vapor of water, and which constitutes 11.4% of the total heat, under the most favorable circumstances.

Mr. Kent states that such a loss is not experienced in Siemen's system, where the products of combustion of Siemen's gas $(CO + N)$, are $CO_2 + 2N +$ excess of air.

Dr. Henry Wurtz shows that 37.5 pounds of anthracite will furnish 1000 cubic feet of water gas, which has a heating power of 311 Fahr. units per cubic foot; or 311000 units per thousand cubic feet.

A pound of coal, therefore, furnishes a volume of gas which, when burned, yields $\frac{311000}{37.5} = 8229.33$ heat units. The coal itself, when burned, should yield at least 13000 units. The thermic value of the water gas is, therefore, only

$$\frac{8229.33 \times 100}{13000} = 63.3\%$$

of that of the coal from which it is produced.

Mr. Wm. A. Goodyear makes the efficiency of water gas 81%.

If 37.5 pounds of coal yield 1000 cubic feet of water gas, a *ton* of coal should yield $\frac{2240}{37.5} = 59733$ cubic feet.

4. *Natural Gas; Its Fuel Value.*—Prof. J. P. Lesley, State Geologist of Pennsylvania, estimates:

7.5 *c. f.* gas = 1 lb. coal.
25. " " = 1 "

whence 1 lb. gas is equivalent to $3\frac{1}{3}$ lbs of coal. Natural gas is oderless.

Mr. S. A. Ford, Chief Chemist of the Edgar Thomson Steel Works, reports:

Heat-units in 1000 *c. f.* of gas = 210,069,604.
" " " 38 lbs. " = 210,069,604.
" " " 38 lbs. *C.* = 139,398,896.

One pound of gas = 26.32 cubic feet

Therefore 1000 *c. f.* of gas = 38 lbs., is equivalent to 57.25 pound of carbon; or 1 lb. of gas is equivalent to 1.5 lbs. of carbon.

Similarly, 1000 *c. f.* = 38 pounds of natural gas is equivalent to 62 97 pounds of coke (90 % *C*), 54.4 pounds of bituminous coal and 58 4 pounds of anthracite coal. These are average values. Per pound of gas, the comparison is as follows:

1 pound natural gas	= 1.5	pounds	carbon.
1 " " "	= 1.66	"	coke.
1 " " "	= 1.43	"	bituminous coal.
1 " " "	= 1.54	"	anthracite "

5. *Relative Values of Natural Gas and Coal.*—

"*Practical Electricity*," Aug. 1887, *pp* 19-20, says, in substance:

55.5 pounds of Pittsburg coal (bituminous) yields the same number of heat units as 1000 *c f.* of natural gas. With coal at $1.20 per short ton (5 cents per bushel), 1000 *c. f.* of gas would be worth $\frac{55.4 \times 120}{2000} = 3.32$ cents.

The tests of the Westinghouse Air-Brake Co. have shown that 1.18 *c. f.* of natural gas evaporated one pound of water, from a temperature of 190° Fahr., in a boiler in which one pound of coal evaporated 10.38 pounds of water.

Thus one pound of coal = $1.18 \times 10.38 = 12.25$ *c. f.* of gas; or 1000 *c f.* of gas = $\frac{1000}{12.25}$ 81.63 pounds of coal. It would therefore appear that 38 pounds of gas = 81.63 pounds of coal; or one pound of gas = 2.15 pounds of coal. The small relative value of the coal, in this case, is thus accounted for:

"This results from the expenditure of heat necessary to raise the solid fuel to the gaseous state, which must be done before combustion can take place."

6. *New Method of Making Water Gas.*—This method, which was recently communicated to the French Academy of Sciences, seems to be important. The process is as follows:

A jet of superheated steam is directed into a retort filled with incandescent coke. The O unites with C, forming CO, and H is liberated. The gases are then led to a second retort, filled with some refractory substance, at a red heat, exposing a glowing surface to the gases. At the same time superheated steam is introduced. This siezes upon the carbonic oxide forming CO_2 and more H is liberated.

A milk of lime bath removes the CO_2 and pure H is led to a reservoir.

One ton of coke thus produces about 69000 *c.f.* of gas; or about eleven times as much as the product of illuminating gas from a ton of coal.

This reduces the cost to little, if anything, more than that of natural gas, when the difficulty of controling the latter is considered.

Inventors are at work devising means for carburetting the H, and Boulogne Sur Seine is to be lighted with the gas.

7. *Tests of the Efficiency of the Ward Boiler by a Board of Naval Engineers.*—These tests were made in 1884, by a Board consisting of Chief Engineers B. F. Isherwood, Theo. Zeller and Geo. P. Hunt. They are referred to here because of the variable conditions under which they were made, because of the precision with which the various data were determined and because of the very important and instructive results deduced.

In regard to the boiler, it will be sufficient for our purpose to say that the area of its grate was 3.687 square feet, and that its heating surface was 121.6 square feet; that the duration of the experiments—13 in number—was, as a rule, 24 hours; that the rate of combustion varied from 7.895 pounds to 62.517 pounds per square foot of grate per hour, and that the steam pressure, above zero, in the several experiments ranged from 16 to 255.78 pounds; also that the type of boiler was one in which the water and steam are entirely surrounded by the products of com-

bustion, and in which the loss of heat by radiation was practically constant; it was at least independent of the temperature of the water and steam.

The following table will show some of the more important fundamental data and results of the tests:

Steam pr. above at.	Lbs. Coal per sq. ft. grate per hour.	Lbs. of Steam per pound of Coal.*	Kind of Coal.	Kinds of Draught.
0.00	14.23	8.58	Semi-bituminous	Natural } Slightly reduced by damper.
0.00	12.97	9.00	" "	"
0.00	13.27	8.40	" "	"
160.72	62.52	6.34	" "	Steam blast in chimney
0.00	7.90	8.59	Anthracite	Natural
16.82	15.51	8.28	"	Steam blast in chimney
50.62	16.41	7.83	"	" " " "
54.87	16.67	7.71	"	" " " "
99.53	16.62	7.78	"	" " " "
100.67	16.64	7.73	"	" " " "
142.97	16.47	7.66	"	" " " "
194.79	16.38	7.40	"	" " " "
240.96	16.36	7.26	"	" " " "

The height of the chimney was 24 feet above the grate.

The greater rate of combustion of semi-bituminous coal, with natural draught, will be observed. The report of the trials states that, with a chimney 64 feet high, the rates of combustion, with natural draught, would have been, for anthracite, 10.28 pounds, and for semi-bituminous, 19.51 pounds per square foot of grate per hour.

This is an important consideration where boiler space is limited.

8. *Air Supplied, per pound of Coal.*—In the tests just referred to the supply of air to the furnace was measured by means of a delicate anemometer. The means of the determinations in trials *A*, *B*, *C* and *D*, show that the supply of air was 1.987 times that chemically necessary to combine with the combustible elements of the coal; also, that in the other tests, where anthracite was burned, the supply of air was 2.095 times that chemically necessary. The less supply in the case of the semi-bituminous coal is stated to be due to the greater quantity of oxygen in that coal. The conclusion is reached, and is for the first time stated, that a double supply of air, or oxygen, is essential to perfect combustion.

* From feed water at 212° Fahr.

The report of Chief Engineer Isherwood and his associates, contains a statement of the results of the analyses of the products of combustion, drawn from the chimney, by Professor Walter R. Johnson, for the Navy Department, in the years 1842–1843, as follows:

Kind of Coal.	*Per centum of free O. in the gases of Combustion.*
Pennsylvania anthracite, from five localities,	10.929
" semi-bituminous coal, from four localities,	10.547
Maryland " " " " five "	10.166
Virginia " " " seven "	10.843
British " " " three "	9.953
Two specimens of coke, one natural and one artificial,	10.865
One specimen of dry pine wood	10.213
Mean	10.569

In the example, Art. 25, where provision is made for a double supply of air, the oxygen weighs $2\,(2.442 + 0.077) = 2.519 \times 2 = 5.038$ pounds, and the products of combustion 22.8 pounds. The oxygen constitutes $\frac{5.038 \times 100}{22.8} = 22.1\ \%$.

The oxygen in the surplus of air is one-half of this, and therefore constitutes 11.05 % of the entire products. The fact that Professor Johnson found nearly this quantity of free oxygen always present in the gases of combustion certainly indicates that about a double supply of air was admitted to the furnace in each case, which also was true in the case of the experiments under consideration. Does it necessarily follow that a double supply of air is essential to perfect combustion?

As bearing upon this point, the results of certain experiments of Bunsen are given in the report, as follows: "He discovered that when carbonic oxide, or hydrogen, was mixed with the exact proportion of oxygen necessary for combustion, and to this mixture a non-combustible gas—nitrogen—was added in such quantity that, when the whole mass was fired, its temperature of combustion did not exceed about 2000° Fahr., exactly one-half of the carbonic oxide, or of the hydrogen, was burned; the remaining half having lost its power of combining, and consequently remaining unconsumed. Hence, in order to consume the whole of the carbonic oxide, or hydrogen, twice the chemically necessary quantity of air or oxygen must be supplied."

The report then says: "This is precisely what occurs in the furnace of a steam boiler, in which carbon and hydrogen are burned under conditions which permit the first to be converted into carbonic acid gas, and the last into steam. When one-half of the oxygen admitted is consumed, or combined to saturation, the whole of the accompanying nitrogen of the air remaining inert, the temperature of the gases of combustion in the furnace is about 2000° Fahr., and further combustion, that is, combination of the oxygen with the combustible, ceases."

This conclusion, if correct, under the conditions of boiler practice, is a most important one.

9. *Effect of Increase of Pressure, and Temperature, of Steam, Upon the Economical Development of Power.*—We have shown that high pressures insure greater economy in the development of steam power than low pressures. This fact has long been known; but, so far as is known, no limit of pressure has been fixed.

This question is very fully discussed in the report of the Naval Board upon the experiments with the Ward boiler. Attention is called to the fact that the total heat of steam increases with its pressure, and that, as the pressure and temperature of the steam are increased, the mean difference between the temperatures of the products of combustion and of the boiler is diminished. This being the case it follows, necessarily, that less heat, per pound of coal, is given up to the boiler, when higher pressures are carried, and therefore that the rate of evaporation, per pound of coal, must be less, with high-pressures and temperatures, than with low pressures and temperatures.

Again, attention is called to the fact that, since the volume of steam diminishes less rapidly than the pressure increases, the products of the pressures by the volumes, must continuously increase with the pressures. Also, that the quotients, obtained by dividing these products by the corresponding numbers of heat units due to the evaporation, which represent the values of a heat-unit, in power, continuously increase with the pressure of the steam.

The question thus resolves itself into a determination of the relative *rates of increase*, of the fuel cost of steam, due to increasing pressure, and of the power due to a unit of steam. Should

these rates be uniform and equal, then there will be no economy in the use of high pressure steam. But should the power value increase more rapidly than the fuel cost, then there will be a corresponding advantage due to the use of high pressure steam. It will of course be understood that *total* power is meant, and not the indicated, nor the net power.

The Naval Board, in discussing this question, constructed an interesting and instructive table, which, with the omission of decimals not essential to our purpose, is as follows:

The first column (p) contains the experimental steam pressures, above zero; the second column (v) contains the volumes of the steam per pound of water under the experimental pressures; the third column ($p\,v$) contains the products of the pressures by the volumes, which represent the works; the fourth column (H-100°) contains the total heat-units required to evaporate a pound of water from and at 100° Fahr., and represents the fuel costs of the powers, or works, in the column ($p\,v$); the fifth column $\left(\frac{p\,v}{H\text{-}100}\right)$ contains the power or work values of a unit of heat under the several conditions; the sixth column contains the quotients obtained by dividing the several values in the fifth column by the *first* value in that column; the seventh column contains the relative experimental evaporations under the several pressures; the eighth column contains the relative economic *gains*, due to increase of pressure, and the ninth column contains the relative economic *losses* due to the increase of pressure.

These results show that there is an economic *loss*, as the pressure increases up to a point between 69 and 114 pounds above zero; and that, above this point, there is an economic *gain* due to an increase in the pressure.

Experiments.	1. p.	2. v.	3. $p.\,v$.	4. $H-100°$	5. $\frac{p\ v.}{H-100°}$	6.	7. Relative evaporations, per lb. of combustible.	8. Economic gains.	9. Economic losses.
F.	31.44	12.650	397.7358	1091.02	0.3646 =	1.0000	1.0000	......	......
G.	65.18	6.339	413.2001	1104.77	0.3740 =	1.0259	0.9593	......	0.0148
H.	69.38	5.982	414.9863	1106.04	0.3752 =	1.0292	0.9578	......	0.0130
I.	114.03	3.799	433.2183	1116.73	0.3879 =	1.0641	0.9416	0.0057	......
J.	115.16	3.767	433.8220	1116.96	0.3884 =	1 0654	0.9420	0.0074	......
K.	157.63	2.860	450.8992	1124.26	0.4011 =	1.1002	0.9276	0.0278	......
L.	209.43	2.253	471.7565	1130.04	0.4179 =	1.1463	0.9074	0.0537	......
M.	255.79	1.916	489.9674	1136.68	0.4311 =	1.1824	0.9032	0.0856	......

It will be observed that the comparisons are with the smallest pressure, and that, in experiment *H*, with a pressure of 69.38 pounds, there was an economic *loss* of 1.3 % as compared with experiment *F*.

Beyond 69.38 pounds there appears to be an economic ***gain***, which, in experiment *M*, with a pressure of 255.79 pounds, amounted to 8.56 %.

The conclusion reached is, that for boilers, where the water and steam spaces are enveloped in the gases of combustion, and where the losses from radiation are sensibly constant, there is neither economic loss nor gain due to pressure, when the steam is carried at about 100 pounds above zero; or at about 85 pounds above the atmosphere—the comparison being with steam at a total pressure of 31 pounds; or 16 pounds above the atmosphere.

In *shell* boilers there is a loss of heat, due to radiation, which increases with the temperature and pressure. In consequence of this fact, such boilers will give results less favorable than those obtained from the Ward boiler. Making allowance for this greater loss in shell boilers, the Board conclude: that at 140 pounds above zero, or 125 pounds above the atmosphere, there will be neither loss nor gain due to pressure, as compared with the pressure of 31 pounds above zero; that below 140 pounds, and down to 31 pounds there will be an economic loss, and that above 140 pounds there will be an economic gain. It being understood, always, that ***total work***, or power, is used in the comparisons.

In order to institute a similar comparison, under the conditions of practice, the Board took an example of a condensing engine in which a vacuum of 25½ inches was supposed to be maintained; in which the back pressure on the piston was taken at 3½ pounds, and the friction of the engine at 1½ pounds; making an aggregate prejudicial resistance of 5 pounds per square inch of piston. The steam was supposed to be expanded six times; giving a mean total pressure of 0.52 times the initial total pressure, and the initial pressure was taken as 3 pounds less than the boiler pressure.

The results are presented in the following table which is taken from the report.

The headings of the columns need no special explanation.

Experiment.	Total Boiler Pressure p.	Initial Cyl. Pressure $p - 3$ lbs.	Mean Total Pressure $0.52(p-3)$	Pr'judicial Resistances per sq. in.	Mean Net Pressure. $0.52(p-3)-5$	Relative Economy. $\frac{0.52(p-3)-5}{0.52(p-3)}$	Relatively to Exp. *F.*
F.	31.44	28.44	14.79	5	9.79	0.6619 =	1.0000
G.	65.18	62.48	32.33	5	27.33	0.8454 =	1.2771
H.	69.38	66.38	34.52	5	29.52	0.8551 =	1.2919
I.	114.03	111.03	57.73	5	52.73	0.9134 =	1.3799
J.	115.16	112.16	58.32	5	53.32	0.9143 =	1.3812
K.	157.63	154.63	80.41	5	75.41	0.9378 =	1.4168
L.	209.43	206.43	107.34	5	102.34	0.9534 =	1.4404
M.	255.79	252.79	131.45	5	126.45	0.9620 =	1.4533

It will be observed that the numbers in the last column increase rapidly with the lower pressures and less rapidly for the higher pressures. Thus, the gain at 209 pounds is 44 %, while at 255 pounds it is only 45.3%. At 157 pounds the gain is 41.7%.

In practice, however, the Board say, that the gains due to high pressures will be much less than are indicated in the table, for the following reasons:

First.—In consequence of steam leakage past valves and piston, which increases with the pressure.

Second.—In consequence of increased external radiation from steam pipe and cylinders, due to increase of temperature.

Third.—In consequence of the increased internal cylinder condensation due to a greater range of temperatures.

Making a proper allowance for these, the report concludes that a boiler pressure of about 100 pounds above zero, or about 85 pounds above the atmosphere, is one *beyond which "no increase of economy attends increase of pressure."*

The gain of about 10 per centum between 100 pounds and 255 pounds, is regarded as not greater than the losses due to the three causes just mentioned.

The Board also call attention to the fact that the weight and cost of boilers are much less, for moderate pressures, than for extremely high pressures.

10. *Coal Consumption, per I. H. P. per Hour.*—An indicated horse-power is reported to be maintained by the expenditure of 1.2 pound of coal per hour, in a tandem, quadruplex-expansion engine, manufactured by Messrs. Rankin and Blackmore, of Greenock, Scotland. Duration of trial, 3 hours. The cylinders

are 12, 17, 24 and 34 inches in diameter, respectively, and the common stroke of their pistons is 24 inches.

The boiler is 11 feet 6 inches in diameter, by 9 feet 6 inches long; its shell is of mild, cold rolled, steel, $1\frac{5}{32}$ inches thick; its joints are butted with double straps, $\frac{15}{16}$ inch thick; its girth seams are double, and its longitudinal seams are triple riveted. The working steam pressure is 180 pounds per square inch. The condensing surface is 618 square feet; or about 50% less than is usual for compound engines.

If 10 pounds of water were evaporated per pound of coal, the steam per indicated horse-power, per hour, was $1.2 \times 10 = 12$ pounds. The explanation of this unprecedented result, if the report be true, is to be found in the fact that the cylinder condensation, from internal radiation, must have been very slight; owing to the slight changes in temperature in the first three cylinders.

11. *Proportions of Locomotive Cylinders.*—These depend upon the weight on the drivers, upon the mean steam pressure and upon the condition of the rail. For the purpose of securing uniformity in locomotive design, the Railroad Master Mechanics' Association of the United States submitted this question to a committee of its members, who recommended, at the annual meeting of the Association, held at St. Paul, Minn., in 1887, the following formula for the diameters of locomotive cylinders:

$$d = \sqrt{\frac{W D}{0.85\, P s C}} \text{ inches;}$$

in which

$P =$ boiler steam pressure, in pounds.
$d =$ diameter of cylinder, in inches.
$s =$ stroke of piston, in inches.
$D =$ diameter of drivers, in inches.
$W =$ weight on the drivers, in pounds.

$$\frac{1}{C} = \text{coefficient of adhesion,} \left\{ \begin{array}{ll} \frac{1}{4} & \text{for passenger engines,} \\ \frac{1}{4.25} & \text{" freight "} \\ \frac{1}{4.5} & \text{" switching "} \end{array} \right.$$

$0.85\, P$ is taken as the normal mean effective cylinder pressure, in pounds per square inch.

$\frac{1}{C}$ is taken at $\frac{1}{4}$ for a *dry rail*, and it is used for passenger

engines, because they work up to 85% of the boiler pressure only for a limited portion of their mileage.

$\frac{1}{C}$ is taken at $\frac{1}{4.25}$ for freight engines, because they more frequently work up to 85% of their boiler pressures.

$\frac{1}{C}$ is taken at $\frac{1}{4.5}$ for switching engines, because they are liable to work at full stroke, and on greasy rails.

Derivation of the Formula.—The work of the drivers, per revolution, will be the product of their common circumference by their adhesion to the rails; or,

$$\text{Work} = \frac{W \pi D}{C}. \quad \ldots\ldots\ldots\ldots (a)$$

The work of the steam, in the cylinders, per revolution, will be the mean pressure, on both, multiplied by twice the stroke of the pistons. Thus,

$$\begin{aligned} \text{Mean pressure on one piston} &= 0.85\,P \times \tfrac{1}{4}\pi d^2 \\ \text{" \quad " \quad " both pistons} &= 0.85\,P \times \tfrac{1}{2}\pi d^2. \\ \text{Twice the stroke} &= 2s. \end{aligned}$$

Hence,

$$\begin{aligned} \text{Work} &= 0.85\,P \times \tfrac{1}{2}\pi d^2 \times 2s \\ &= 0.85\,P \pi d^2 s. \quad \ldots\ldots\ldots\ldots (b) \end{aligned}$$

Equating (a) and (b) we get

$$\frac{W \pi D}{C} = 0.85\,P \pi d^2 s;$$

whence

$$W D = 0.85\,P d^2 s\,C.$$

$$\therefore d = \sqrt{\frac{W D}{0.85\,P s\,C}}. \quad \ldots\ldots\ldots\ldots (A)$$

This result can be satisfactory only when 0.85 P represents the excess of cylinder pressure over that necessary to overcome the friction of the machinery. In the report of the committee it is understood that by mean effective pressure is meant 0 85 of the boiler pressure, as indicated by the gauge; also that this is the resultant pressure, after the friction of the machinery has been overcome.

12. *Results of the Trial Trip of the U. S. S. Chicago.*—See Art. 103, Ex. 6. The trial trip of the Chicago, in Long Island Sound, in December, 1887, gave the following results, which are kindly furnished by Chief Engineer J. W. Thomson, U. S. N.:

Displacement, tons	4546.
Mean draught of water, feet	19.
Mean revolutions of screws, per minute	69.3
Mean steam pressure, above atmosphere	84.8
Mean indicated horse-power	5083.79
Mean speed, for 6 hours, knots	15.1
Max. steam pressure, pounds	90.
Max. revolutions	71.
Max. speed, knots	16.35
Max. indicated horse-power	5248.42

Deductions.—The diameter of the screws being 15.5 feet and their mean pitch $\left(\frac{24.578 + 24.436}{2}\right)$, being 24.507 feet, the advance due to the mean pitch and mean revolutions, was

$$24.507 \times 69.3 = 1698.335$$

feet per minute; while the actual advance of the vessel was

$$\frac{15.1 \times 6086}{60} = 1531.643$$

feet per minute.

The mean slip of the screws was, therefore,

$$100 \times \frac{1698.335 - 1531.643}{1698.335} = 9.81\ \%.$$

This small slip is due to the employment of *two* screws, in the place of the ordinary single screw.

In order to determine the speed due to the mean displacement and power we have (see tables for speed, power, and displacement),

$$Log.\ C = log.\ I.\ H\text{-}P - log.\ D^{\frac{2}{3}}.$$

$$\begin{array}{lll} Log.\ Ind.\ H\text{-}P & (5083.79) = & 3.70586 \\ Log.\ D^{\frac{2}{3}} & (4546^{\frac{2}{3}}) \ = & \underline{2.43840} \\ Log.\ C & (18.47+) = & 1.26746 \end{array}$$

This value of C, it will be observed, corresponds, exactly, with that due to a speed of 15.1 knots.

For the maximum power and speed we have

$$\begin{array}{lll} Log.\ I.\ H\text{-}P. & (5248.42) & = 3.72015 \\ Log.\ D^{\frac{2}{3}} & (4546^{\frac{2}{3}}) & = 2.43840 \\ \hline Log.\ C & (19.13) & = 1.28175 \end{array}$$

This value of C corresponds to a speed of 15.3 knots, which is a knot less than the reported maximum speed.

13. *The Spanish Cruiser, Regina Regente.*—This vessel, built by Messrs. George and James Thompson, of Clydebank, has the following dimensions, power, etc.

Length, between perpendiculars, feet..........	307.
Beam....................................	50½
Draught of water............................	20.
Displacement, normal, tons....................	4800.
" deep load, tons.................	5600.
" coefficient of...................	0.564
Indicated horse-power.........................	12000.
Pressure of forced draught, inches of water....	1.

She is provided with two triple-expansion engines and twin screws.

The contract requirements were as follows:

Speed, 4 runs over the mile, 20.5 knots; to be continued for two hours, after the four runs.

Speed, to be maintained for six hours, continuously, 18.5 knots, without artificial draught.

The vessel must also be able to steam 5700 knots, on 500 tons of coal, at some speed, greater than 10 knots, to be chosen by the builders.

On the official trial, the following were the results:

Mean steam pressure, pounds................	140.
Mean speed, knots..........................	20.6
Mean indicated horse-power....	11500.
Mean displacement, tons.....................	5000.

With natural draught, the speed was 18.68 knots and the mean revolutions, 94.75 per minute.

With forced draught, the average number of revolutions was 105.5.

A run of 12 hours showed that, at a speed of 11.6 knots, the vessel could steam 5900 knots on 500 tons of coal.

This vessel turns 180° in one minute and twenty-three seconds; and 360° in two minutes and fifty-five seconds, with a radious of 175 yards. Area of rudder, 230 square feet.

The normal displacement of 4800 tons, and the contract speed of 20.5 knots require, according to Eq. (79) and tables *A* and *B*,

$$\left.\begin{array}{lll} Log.\ C & = & 1.63889 \\ \text{"} \quad D^{\frac{2}{3}} & = & 2.45416 \\ \text{"} \quad H\text{-}P & = & \overline{4.09305} \end{array}\right\} \therefore H\text{-}P = 12390.$$

This is 390 horse-power, or about 3%, greater than the power provided by the builders.

In a similar manner we find that, at the official trial speed of 20.6 knots, and displacement of 5000 tons, there would be required, according to Eq. (79), a power of 12898 horses; while the actual power is reported as 11500 horses; or more than 10 % less.

If the reported power be correct, the above difference is probably to be accounted for by the greater efficiency of the twin screws.

The speed of 11.6 knots requires, according to Eq. (79), a power of 2600 horses. This power, according to the report, can be maintained $\frac{5900}{11.6} = 508.6$ hours, on 500 tons of coal. This means that an indicated horse-power, per hour, is to be obtained at an expenditure of

$$\frac{2240 \times 500}{2600 \times 508.6} = 0.847 \text{ of a pound of coal.}$$

This is incredible; and, with the extraordinary speed reported, it gives rise to a suspicion that the performances of the *Regina Regente* as reported, are, to some extent at least, exaggerated.

14. *The New Russian War-Ship Nakhimoff.*—This ship is modeled after the *Imperieuse* of the English Navy. She is 333 feet long, 61 feet broad, and has a displacement of 7782 tons. She is provided with two triple-expansion engines and—presumably —with twin screws.

During her trial trip in October, 1887, the engines, though of 4000 horse power each, worked up to 4500 horses each, or a total of 9000 horses, and gave a speed of 17.5 knots.

From tables *A* and *B*, deduced from Eq. (79), we get:

$$\left.\begin{array}{rl} Log\ C & = 1.44373 \\ \text{"}\ \ D^{\frac{2}{3}} & = \underline{2.59406} \\ \text{"}\ H\text{-}P & = 4.03779 \end{array}\right\} \therefore H\text{-}P = 10910.$$

The reported power, 9000 horses, is 17.52 % less than this. This difference is to be attributed, partly to the fact that 9000 *indicated* horse-power should give a higher speed than the same *total* power, partly to the greater efficiency of the twin screws, and partly—perhaps to exaggeration.

15. *The New British War-Ships, Orlando and Undaunted.*—These are sister ships, of the belted cruiser type. They were designed by the Admiralty, and their dimensions are as follows: Length, 300 feet; beam, 56 feet; draught, forward, 20 feet; draught, aft, 22 feet; displacement, 5000 tons; coefficient of displacement, 0.5; indicated horse-power, 8500, for a speed of 19 knots.

Each is provided with two horizontal, triple-expansion engines and twin screws.

The diameters of the three cylinders of each engine are 36, 52 and 78 inches, respectively; with piston strokes of 42 inches.

The screws are 14.5 feet in diameter and have a pitch of 18.75 feet.

The condensing surface is 12000 square feet; or 1.41 square feet per horse-power.

Centrifugal circulating pumps are provided for the condensers.

The boilers are four in number, 14.5 feet in diameter and 16.5 feet long; with six corrugated furnaces, 3.67 feet in diameter. The grate and heating surfaces are 540 and 16055 square feet, respectively, and the boiler pressure is 130 pounds.

The machinery was designed for a speed of 19 knots per hour.

During the trial trip of the *Orlando*, on April 19th, 1887, the following were the mean results for a run of four hours, with forced draught:

Mean boiler pressure of steam, pounds......... 129.
Mean *I. H-P.*, starboard engine .... 4313.
" " port " .... 4309.
" " total, ... 8622.
" speed, knots............................ 19.25
" air pressure in stoke-hole, inches of water. 1.25

The *Undaunted*, has since made a speed of 19.4 knots; developing a power of 8602 horses.

On April 13th, 1887, the *Orlando* was tried, with natural draught, and made a speed of 17.25 knots during a run of 4 hours, with an indicated power of 5716 horses. The maximum power, during the 4 hours, was 5856 horses; which exceeded the contract power by 356 horses.

We get, Eq. (79), for a displacement of 5000 tons and 5500 horse-power, a speed of 15.2 knots; and for a speed of 19 knots, we get 10275 horse-power.

The designed power, 8500 horses, is 1775 horses, or 17.41 % less than our formula gives. This difference is almost exactly the same as that in the case of the Russian ship, *Nakhimoff*, and is to be accounted for in the same manner as in that case.

16. *Engines of Modern Trans-Atlantic Steamers.*—The following table contains the dimensions of the cylinders, strokes of pistons and powers, of those engines:

Name of Vessel.	*Diameters of Cyls.* Inches.			*Stroke of Pist'n* Inches.	*I. H-P.*
Alaska......................	68	100	100	72	10000
America.....................	63	91	91	66	7354
Aurania.....................	68	91	91	72	8500
City of Rome...............	43 86	43 86	43 86	72	11890
Etruria.....................	71	105	105	72	14321
Umbria......................	71	105	105	72	14321
Oregon (lost)................	70	104	104	72	13300
Servia	72	100	100	72	10300

The *Alaska*, in May, 1883, made, at that time, the quickest passage, in 6 days 23 hours and 48 minutes.

The *Etruria*, in 1886, made a passage in 6 days and 12 hours, at an average speed of $\frac{2859}{156} = 18.33$ knots per hour.

The *Umbria*, in 1887, made a passage, from Queenstown to Sandy Hook, in 6 days and 4 hours; at an average speed of 19.25 knots, or 22.25 miles, per hour.

The *Etruria*, in December, 1887, made a passage in 6 days and 2 hours; at an average speed of 19.58 knots per hour.

This record is likely to be beaten during the year 1888.

17. *Spanish Torpedo Boats, Azor and Halcon.*—These boats, recently built by Messrs. Yarrow & Co., of Poplar, are exactly alike, and have the following dimensions: Length, 135 feet; beam, 14 feet; draught of water, mean, probably about 3.5 feet; displacement, about 100 tons.

The engines, for twin screws, are of 1550 horse-power, and are of the triple-expansion type.

On the trial of the *Azor*, her speed, for 2.5 hours, with a load of 17 tons, was 24 knots.

The speed of the *Halcon*, with the same load, was 23.5 knots.

Eq. (79) gives 1550 horse-power for 110 tons, and a speed of about 23.5 knots.

18. *New Torpedo Boats for the U. S. Navy.*—These are to have the following dimensions, approximately: Length, 135 feet; beam 15 feet; draught, forward, 2 feet; draught, aft, 5 feet 2 inches; displacement, at load line, about 100 tons.

They are to have "the highest attainable speeds."

Premiums are offered, at the rate of $1500 for each quarter knot, above 23 knots; up to and including 24 knots. $2000 is to be paid for each quarter of a knot, in excess of 24 knots.

A penalty of $4000 is to be exacted, if the speed falls below 22 knots. If the speed falls below 20 knots, the boats may be rejected.

They are to have twin screws, and two pairs of triple-expansion engines; with separate engines for circulating and air-pumps. Bids were opened November 1st, 1887, and the prices were about $82500.

19. *The "Now Then."*—This is a steam yacht, built, in the year 1887, by the Messrs. Herreschoff, of Bristol, R. I., for Mr. Norman L. Munro, of New York. She is said to be the fastest steam yacht, of her type, in American waters. Her dimensions, power, speed, etc , together with those of two English built boats, are as follows:

	Now Then.	*Thorneycroft.*	*Yarrow.*
Length of water line, feet............	81.	147.5	140.
Beam, feet..	10.	14.5	14.
Draught, feet.......................	3.25	3.35	5.33
Displacement, tons.................	20.	——	100.
Indicated horse-power..............	——	——	1400.
Speed, knots.......................	19 +	26.0	24.964
" miles.......................	22.	30.	29.–

The "*Now Then*" ran from Newport to New York, about 155 miles, July 12th, 1887, in 7 hours and 4 minutes. She has triple-expansion engines, and is smaller than either of the other boats. Both of the English boats have compound engines, twin-screws and forced draught. The *Thorneycroft* has water-tube, and the *Yarrow* locomotive boilers. The *Yarrow* carried 151.8 pounds of steam, and made 394 revolutions; the piston speed being 985 feet per minute. Her cylinders are 14.5 and 24.25 inches in diameter, and her piston stroke is 15 inches.

20. *The Buzz.*—This is a new boat, designed by Mr. C. D. Mosher, of Amesbury, Mass. Her dimensions are as follows: Length, 50 feet; beam, 6.5 feet; "water-line," 4.87 feet; depth, 3 feet; draught, forward, 0.75 feet; draught, aft, 1.333 feet; displacement, tons, 3.5; midship section, "circular," 4.5 square feet.

Her screw is of phosphor-bronze, 2.67 feet in diameter and 5 feet pitch. It is of peculiar design, having a lip turning aft, at its periphery.

The hull is said to be so modeled that its stern does not settle when she is running at full speed.

The boiler is of the locomotive type, 32 inches in diameter. It has 250 brass tubes, 5 feet long. Its fire-box is 38 inches long by 24 inches wide; giving a grate area of 9 square feet. The heating surface is 357 square feet; or about 30 times the grate area. Steam pressure carried, 150 pounds.

The draught is forced by a Sturtevant blower, driven by a 4½ inch belt, from the main shaft, and making 5000 revolutions per minute. Anthracite coal, only, is burned.

The cylinders are two in number, 8 by 8 inches, are vertical and directly connected. Clearance, 4%.

The crank-shaft, is of steel, and has three bearings, 2⅝ inches in diameter by 6 inches long.

The crank-pins are hollow, are $2\frac{5}{8}$ inches in diameter, and are $4\frac{1}{8}$ inches long.

The piston-rod is $1\frac{1}{4}$ inch in diameter and the screw-shaft $2\frac{1}{8}$ inches in diameter—with a ball thrust.

The engines make over 600 revolutions, without pounding, and develop a power of over 160 horses.

The weight of the engines is 703 pounds; while the weight of the engines and boiler, with water, is only 4700 pounds.

The *Buzz.* carries 1.5 tons of coal, and can steam 800 miles, at a speed of 10 knots.

At her maximum speed, she makes the "measured mile" in 2 minutes and 8 seconds; corresponding to a speed of 28.12 miles per hour. This is probably the greatest recorded speed, for a boat of this size.

21. *Fast Hudson River Paddle-Wheel Steamers.*—

The *Mary Powell* has long been reputed to be the fastest steamer on the Hudson. Her speed is, however, exceeded by that of the new steamer, *New York*, of the Albany and New York Day Line.

The dimensions, powers, speeds, etc., of these boats are as follows:

	Mary Powell.	*New York.*
Length, water line, feet	286.	301.
Beam, molded, feet	34.25	40.
Draught of water, feet	6.	6.
Depth, feet	11.5	12.25
Displacement, tons	800.	1051.
Tonnage	——	1552.
Indicated horse-power	1590.	3850.
Speed, knots	17.2	21.+
Coefficient of displacement	0.475	0.528

The *New York*, on her trial trip, with 40 pounds of steam, made 24 miles = 20.82 knots, per hour. She is reported to have made a speed of 23 miles, against wind and tide, with 37 to 39 pounds of steam.

Eq. (79), gives, for the *Mary Powell*, a speed through the water, of 15.2 knots.

For the *New York*, with a displacement of 1051 tons, and an indicated power of 3850 horses, we get a speed of 19.5 knots; which is something more than a knot less than her reported speed.

The *New York* has feathering paddles, placed abaft the centre of length. The wheels are 30 feet 2 inches in diameter, and each has 12 curved steel buckets, 3.75 feet wide and 12.5 feet long. The wheels are over-hanging.

The engine is of the overhead beam type, with the steam cylinder 75 inches in diameter, and a piston stroke of 12 feet. It has a Stevens cut-off, and a surface condenser.

There are 3 return-flue boilers, 9.25 feet in diameter, 33 feet long and 11 feet wide on the front.

The grate surface is $3 \times 76 = 228$ square feet.

Steam is to be carried, if necessary, at 50 pounds; and the power, with forced draught, is 3850 horses.

The *New York* ran from 22nd St., New York, to Yonkers, 14.5 miles, in 36 minutes; or at an average speed of 24.17 miles per hour. Her revolutions were 28, with a steam pressure of 40 pounds.

If the diameter of the centres of the buckets be 26 feet, 28 revolutions give 2296 feet per minute as the movement of the centres of the paddles. The movement of the steamer, over the ground, was $24.17 \times 88 = 2126.96$ feet per minute.

This indicates a slip of 169.04 feet, or 7.38%; which is small, and suggests that the steamer ran *with the tide.*

Taking the tide at 2 miles per hour or 176 feet per minute, the actual movement of the steamer, through the water, would appear to have been $2126.96 - 176 = 1950.96$ feet per minute; and the slip $= 2296 - 1950.96 = 345.04$ feet per minute, or 15%.

This is a reasonable, and probable, slip, and points, almost conclusively, to the fact that the steamer ran with the tide.

22. *Fast English Paddle-Wheel Steamers.*—The *Queen Victoria*, and the *Prince of Wales*, two sister paddle-wheel steamers, with feathering paddles, built by the Messrs. Elder, for the Liverpool and Isle of Man service, have the following dimensions, power, etc.:

Length, feet	340.
Beam, "	39.
Depth, "	24.
Gross tonnage	1500.
Displacement, tons, say	1200.
Indicated horse-power	6000.
Speed, *Queen Victoria*, knots	24.5

The *Queen Victoria* has since run 24 knots at a rate of 22.6 knots, or 26.1 miles per hour.

The *Prince of Wales* ran from Greenock to Liverpool, 240 miles, in 9 hours and 26 minutes, at the rate of 22.5 knots, or 25.6 miles per hour.

The engines of these steamers are compound, two cylinder, diagonal, direct-acting and condensing. The *h. p. cyl.*, 61 inches in diameter, is placed above the *l. p. cyl.*, which is 112 inches in diameter.

Forced draught is employed. The shafts and crank-pins are of steel, and are hollow. The paddle arms and feathering floats are also of steel.

Eq. (79), gives, for a displacement of 1200 tons, and for an indicated power of 6000 horses, a speed of about 22 knots.

23. *The Steamer Meteor.*—This steamer, recently built on the Clyde, by Messrs. J. and G. Thomson, with triple expansion engines, in August, 1877, ran from London to Leith, wharf to wharf, in 27 hours 45 minutes. The run from Gravesend to Leith, 475 nautical miles, was made at an average speed of 18.5 knots, or 21.3 miles, per hour.

As compared with her predecessor, the *Ionia*, of exactly the same form and dimensions, but fitted with compound engines, the *Meteor* is 1.2 knots faster; her engines develop 50% more power, although only 16% heavier. Her hull is 5% lighter; and thus heavier engines are carried on the same draught of water.

We here have an illustration of the advantage to be expected from the substitution of the triple-expansion, for the compound engine.

24. *New Vessels for the U. S. Navy.*—The following table contains such information as is attainable in relation to the new vessels, recently built, or now in process of construction, for the navy of the United States:

Designation.	Name or Type.	Type of Engines.	Displacement in Tons	Speed in Knots	Ind. H. Power. Designed and Trial.	By Eq. (79).
	Chicago, 1882.........	C'mp'nd	4500	15	5000	5305
	" Trial.......	"	4546	15.1	5083.8	
	Boston, 1882...........	"	3000	15.	3500	3775
	" Trial..........	"	?	13.8*	3779.6	
	Atlanta, 1882..........	"	3000	15.	3500	3775
	" Trial.........	"	?	15.5	3356.4	
	Dolphin, 1882.........	"	1500	15.	2300	2360
	" Trial........	"	1485	15.5	2240	
Cruiser, No. 2....	Charleston,[1] 1885......	"	3730	18.9	7650	8300
" " 3....	Baltimore,[1] 1886.......	Trip. ex.	4413	19.	9000	9440
" " 1....	Newark, 1885..........	"	4083	18.	8500[2]	7960
	Gunboat, No. 1, 1885...	"	1700	16.	3000[2]	3085
	" " 2, 1885...	C'mp'nd	870	13.	1300	1135
Armored Cruiser.	Cruiser, No. 1, 1886....	Trip. ex.	6648	17.	9000	9075
Battle-Ship......	Battle-Ship,[1] No. 2, 1886	"	6300	17.	8600	8750
	Puritan................	C'mp'nd	6060	13.2	3700	4275
	Amphitrite............	"	3815	11.	1600	1950
	Monadnock.	Not de'd	3815	?	....	
	Terror.................	C'mp'nd	3815	11.	1600	1950
	Miantonomah....	"	3815	11.	1600	1950
	Dynamite Gun Cr., 1886	Trip. ex.	700	20.	3500	3200
Cruiser, No. 4. ..	Steel Cruiser, 1887.....	"	4400	19.	9500	9425
" " 5....	" " "	"	4083	19.	8000	8950
Gunboat, No. 3..	" Gunboat, "	"	1700	16.	3000[2]	3085
" " 4..	" " "	"	1700	16.	3000[2]	3085

It is exceedingly difficult, in such cases, to ascertain the exact designs, the conditions under which trials are made, and the results of such trials. The information contained in the foregoing table has been obtained, partly from technical journals, partly from the Navy Department, and partly from contractors who are constructing machinery from their own designs.

25. *Unarmored Steam-Ships of the British Navy.*—The following table contains the lengths, breadths, displacements, indicated powers and speeds, both actual and estimated—by Eq. (79)—of the unarmored steam vessels of the British navy, built between 1868 and 1880.

The list is taken from the *Encyclopædia Britanica* and can probably be depended upon as correct.

We have added the final column, which contains the speeds as estimated by Eq. (79), by the aid of tables *A* and *B*.

* Low speed due to foul bottom. Ship had not been docked for a year.

1. Machinery and ship of English design.

2. No speed guaranteed. The indicated powers of the three gunboats, of 1700 tons, is understood, in the Bureau of Engineering, to be 3300.

The speeds in the column headed "actual," are represented by being either "ascertained or estimated."

The *armored* ships are omitted from the table, because their speeds are not given, and because, for that reason, no comparison of the speeds with those given by Eq. (79), is possible.

UNARMORED SHIPS OF THE ENGLISH NAVY.—STEAMERS.

No.	Name.	Dimensions and Powers.				Speeds.	
		Length.	Br.	Disp.	*I. H-P.*	Actual.	Est'd.
1	Inconstant	337′ 4″	50′ 3″	5780	7360	16.20	16.75
2	Raleigh	298	49	5200	6160	15.32	15.85
3	Shah	334′ 8″	52	6250	7480	16.20	16.28
4	Mersey (*T*)*	300	46	3550	6000	17.00	17.08
5	Severn (*T*)*	"	"	"	"	17.00	"
6	Thames (*T*)*	"	"	"	"	17.00	"
7	Active	270	42	3080	4010	14.97	15.23
8	Amethyst	220	37	1970	2140	13.24	13.42
9	Bacchante	280	45′ 6″	4130	5250	15.00	15.61
10	Boadicea	280	45	4140	5290	14.70	15.75
11	Briton	210	36	1860	2150	13.13	13.7
12	Canada*	225	44′ 6″	2380	2300	13.00	13.22
13	Caroline*	200	38	1420	1500	13.00	12.83
14	Carysfort*	225	44′ 6″	2380	2300	13.00	13.22
15	Calliope*	235	44′ 6″	2770	3000	13.75	14.05
16	Calypso*	"	"	"	"	13.75	14.05
17	Cleopatra	225	"	2380	2300	13.00	13.22
18	Champion	225	"	"	"	13.00	13.22
19	Comus	"	"	"	"	13.00	13.22
20	Conquest	"	"	"	"	13.00	13.22
21	Constance*	"	"	"	"	13.00	13.22
22	Cordelia	"	"	"	"	13.00	13.22
23	Curacoa	"	"	"	"	13.00	13.22
24	Diamond	220	37	1970	2150	12.56	13.50
25	Dido	212	36	1760	2520	13.50	14.72
26	Druid	220	36	1860	2270	12.90	13.99
27	Eclipse	212	36	1760	1950	12.90	13.38
28	Emerald	220	40	2120	2170	13.20	13.30
29	Encounter	220	37	1970	2130	13.19	13.45
30	Euryalus	280	45′ 6″	4140	5270	14.72	15.70
31	Garnet	220	40	2120	2000	13.00	12.93
32	Heroine	200	38	1420	950	13.10	14.13
33	Hyacinth	"	"	"	"	13.10	14.13
34	Juno	200	40′ 4″	2240	1380	10.87	11.03
35	Modeste	220	37	1970	2180	12.79	13.52
36	Opal	220	40	2120	2120	13.37	13.20
37	Plyades*	200	38	1420	1500	13.00	12.83
38	Rapid	"	"	"	"	13.00	12.83
39	Rover	280	43′ 6″	3460	4960	14.53	16.00
40	Royalist*	200	38	1420	1500	13.00	12.83
41	Ruby	220	40	2120	1830	12.28	12.45
42	Saphire	220	37	1970	2360	13.58	13.99
43	Satellite	200	38	1420	950	13.10	14.13
44	Tenedos	212	36	1760	2040	12.80	13.63
45	Thalia	200	40′ 4″	2240	1600	11.14	11.60

UNARMORED SHIPS—CONTINUED.

No.	Name.	Dimensions and Powers.				Speeds.	
		Length.	Br.	Disp.	*I.H-P.*	Actual.	Est'd.
46	Thetis	220	36	1860	2270	13.39	14.00
47	Tourmaline	220	40	2120	1970	12.62	12.85
48	Turquoise	"	"	2120	1990	12.32	12.85
49	Volage	270	42	3080	4530	15.08	15.93
50	Iris (*T*)	300	46	3730	7000	18.00	17.80
51	Mercury, 1878 (*T*)	"	"	"	"	18.00	17.80
52	Leander (*T*)*	"	"	3748	5000	16.00	15.78
53	Phaeton (*T*)*	"	"	"	"	16.00	15.78
54	Arethusa*	"	"	3748	"	16.00	15.78
55	Amphion (*T*)*	300	46	3750	5000	16.00	15 78
56	Scout(*T*.)*	220	34	1430	3200	16.00	16.75
57	Albatross	160	31′ 4″	940	840	10.51	11.39
58	Alert	160	31′ 11″	1240	310	7.68	7.00
59	Cormorant	170	36	1130	950	11.31	11.39
60	Daring	160	31′ 4″	940	920	10.64	11.80
61	Dragon	170	36	1140	1010	11.52	11.65
62	Dryad	187	36	1620	1570	11.87	12.60
63	Egeria	160	31′ 4″	940	1010	11.30	12.21
64	Espiègle*	170	36	1137	900	11.50	11.16
65	Fantome	160	31′ 4″	940	970	11.06	11.51
66	Fawn	160	31′ 10″	1050	480	9.36	8.90
67	Flying Fish	160	31′ 4″	940	840	10.96	11.39
68	Gannet	170	36	1130	900	11.53	11.19
69	Kingfisher	170	36	1130	900	11.60	11.19
70	Miranda	170	36	1130	900	11.50	11.19
71	Mutine*	170	36	1137	900	12.00	11.16
72	Osprey	170	36	1130	1010	11.20	11.57
73	Pegasus	170	36	1130	970	11.47	11.42
74	Pelican	170	36	1130	1060	11 60	11.90
75	Penguin	170	36	1130	760	10.00	10.45
76	Sappho	160	31′ 4″	940	880	10.59	11.60
77	Wild Swan	170	36	1130	800	10.35	10.63
78	Enchantress	220	28′ 2″	1000	1320	14.02	13.31
79	Helicon	220	28′ 2″	1000	1610	13.05	14.35
80	Vigilant	220	28′ 2″	1000	1810	13.27	14.99
81	Hecla	391′ 7½″	38′ 9¼″	6400	1760	11.70	9.40
82	Vesuvius (*T*)	90	22	244	390	9.71	12.00
83	Bittern (*T*)	170	29	805	850	10.70	11.92
84	Algerine	157	29′ 6″	835	810	10.50	11.60
85	Flirt (*T*)	155	25	603	530	9.68	10.70

26. *Tables, Facilitating the Determination of Power and Speed.*—Eq. (79) is

$$H\text{-}P = C D^{\frac{2}{3}};$$

in which $H\text{-}P$ is the horse-power, D the displacement in tons

*Building in 1883. Those marked (*T*) have twin screws.
The "Enchantress," "Helicon" and "Vigilant" have paddles.

and C a constant, depending upon the speed, whose value is

$$C = V(0.1552 + 0.0046846\,V^2);$$

V being the speed, in knots per hour.

The values of C have been calculated for speeds, varying by a single knot, from 10 to 20 knots. These values were laid off, as ordinates, at equal intervals, along an axis of abscissæ.

A fair curve was then drawn through the extremities of the ordinates. Afterward, the intervals between these ordinates were each subdivided into ten equal parts, and other ordinates erected at the several points of subdivision. Thus, by construction, were found values of C for each tenth of a knot.

Subsequently, the values of C were found between 20 and 25 knots, for intervals of a quarter of a knot.

From Eq. (79), we have

$$Log.\ H\text{-}P = log.\ C + log.\ D^{\frac{2}{3}}, \quad \ldots\ldots\ldots\ldots\ldots (a)$$

and

$$Log.\ C \quad = log.\ H\text{-}P - log. D^{\frac{2}{3}}. \quad \ldots\ldots\ldots\ldots\ldots (b)$$

If, then, the speed, V, and the displacement, D, be given, we readily find the power, $H\text{-}P$, by means of (a).

If, the power, $H\text{-}P$, and the displacement, D, be given, we find C, and then V, by means of (b).

Table A contains, in the first column, numbers, from 1000 to 10550, with intervals of 50; in the second column, the logarithms of the numbers in the first column, and in the third column, logarithms which, in each case, are two-thirds of those in the second column.

The numbers in the first column may represent either power or displacement; if they represent *power*, the logarithms in the second column are to be used; if they represent displacement, the logarithms in the third column are to be used.

Table B, contains, in the first column, values of V, with tenth of a knot intervals, from 10 knots to 20 knots, and quarter of a knot intervals, from 20 to 25 knots; in the second column, cor-

responding values of C, and in the third column, logarithms of C.

Example.—Given, a displacement of 5000 tons and a power of 7500 horses, what speed may be expected?

Use (b). In Table A, we find

$$\begin{aligned} Log.\ 7500 &= 3.87506 \\ \text{"}\quad 5000^{\frac{2}{3}} &= \underline{2.46598} \\ \text{"}\quad C &= 1.40908 \end{aligned}$$

In Table B, we look for this logarithm and find, opposite $V =$ 17 knots, 1.40909.

We may therefore expect a speed of 17 knots per hour.

Example 2.—Given a displacement of 5000 tons, what power will be required to give the vessel a speed of 16 knots?

Use (a). In Table B, we find

$$\begin{aligned} & Log.\ C_{11} &&= 1.33586; \\ \text{in Table } A, \quad & \text{"}\quad 5000^{\frac{2}{3}} &&= \underline{2.46598;} \\ \therefore & \text{"}\quad H\text{-}P &&= 3.80184 \end{aligned}$$

In Table A, we find the next less logarithm to be 3.79934; and the next greater logarithm, 3.80277. These logarithms correspond to powers of 6300 and 6350 horses, respectively.

We may therefore put the power at 6335 horses.

In using these tables, it is to be remembered, *always*, that there is a distinction to be made between the *total*, and the *indicated* horse-power; and that the latter is, usually, from 5% to 8% *less* than the former.

It is to be remembered, also, that the constants, in Eq. (78), were deduced from the performances of U. S. vessels whose speeds did not exceed 10 knots per hour; and therefore that the formula is perhaps less reliable, for high speeds, than it would be were its constants calculated from the performances of modern high speed vessels.

TABLE A.

VALUE OF *D*, *H. P.*, *log.* $D^{\frac{2}{3}}$ AND *log. H. P.* TO BE USED IN THE DETERMINATION OF POWER AND SPEED BY MEANS OF EQUATIONS (78) AND (79).

1 D. or H-P	2 Log. H-P.	3 Log. $D^{\frac{2}{3}}$	1 D. or H-P	2 Log. H-P.	3 Log. $D^{\frac{2}{3}}$
500	2.69897	1.79931	2900	3.46240	2.30827
550	2.74036	1.82696	2950	3.46982	2.31321
600	2.77815	1.85210	3000	3.47712	2.31808
650	2.81291	1.87528	3050	3.48430	2.32287
700	2.84510	1.89673	3100	3.49136	2.32757
750	2.87506	1.91671	3150	3.49831	2.33221
800	2.90309	1.93539	3200	3.50515	2.33677
850	2.92942	1.95295	3250	3.51188	2.34126
900	2.95424	1.96949	3300	3.51851	2.34568
950	2.97772	1.98515	3350	3.52504	2.35003
1000	3.00000	2.00000	3400	3.53148	2.35432
1050	3.02199	2.01413	3450	3.53782	2.35855
1100	3.04139	2.02760	3500	3.54407	2.36271
1150	3.06070	2.04047	3550	3.55023	2.36682
1200	3.07918	2.05279	3600	3.55630	2.37087
1250	3.09691	2.06461	3650	3.56229	2.37486
1300	3.11394	2.07596	3700	3.56820	2.37880
1350	3.13033	2.08689	3750	3.57403	2.38269
1400	3.14613	2.09742	3800	3.57978	2.38652
1450	3.16137	2.10758	3850	3.58546	2.39031
1500	3.17609	2.11739	3900	3.59106	2.39404
1550	3.19033	2.12689	3950	3.59660	2.39773
1600	3.20412	2.13608	4000	3.60206	2.40137
1650	3.21748	2.14499	4050	3.60745	2.40497
1700	3.23045	2.15363	4100	3.61278	2.40853
1750	3.24304	2.16203	4150	3.61805	2.41203
1800	3.25527	2.17018	4200	3.62325	2.41550
1850	3.26717	2.17811	4250	3.62839	2.41893
1900	3.27875	2.18584	4300	3.63347	2.42231
1950	3.29003	2.19336	4350	3.63849	2.42566
2000	3.30103	2.20069	4400	3.64345	2.42897
2050	3.31175	2.20784	4450	3.64836	2.43224
2100	3.32222	2.21481	4500	3.65321	2.43547
2150	3.33244	2.22163	4550	3.65801	2.43867
2200	3.34242	2.22828	4600	3.66276	2.44184
2250	3.35218	2.23479	4650	3.66745	2.44497
2300	3.36173	2.24115	4700	3.67210	2.44807
2350	3.37107	2.24738	4750	3.67669	2.45113
2400	3.38003	2.25335	4800	3.68124	2.45416
2450	3.38917	2.25944	4850	3.68574	2.45716
2500	3.39794	2.26496	4900	3.69020	2.46013
2550	3.40654	2.27103	4950	3.69461	2.46307
2600	3.41497	2.27665	5000	3.69897	2.46598
2650	3.42325	2.28216	5050	3.70329	2.46886
2700	3.43136	2.28758	5100	3.70757	2.47171
2750	3.43933	2.29289	5150	3.71181	2.47454
2800	3.44716	2.29811	5200	3.71600	2.47733
2850	3.45484	2.30322	5250	3.72016	2.48011

TABLE A—Continued.

1 *D.* or *H-P.*	2 *Log. H-P.*	3 *Log.* $D^{\frac{2}{3}}$	1 *D.* or *H-P.*	2 *Log. H-P.*	3 *Log.* $D^{\frac{2}{3}}$
5300	3.72428	2.48285	7950	3.90037	2.60024
5350	3.72835	2.48557	8000	3.90309	2.60206
5400	3.73239	2.48826	8050	3.90580	2.60386
5450	3.73640	2.49093	8100	3.90848	2.60566
5500	3.74036	2.49358	8150	3.91116	2.60744
5550	3.74429	2.49620	8200	3.91381	2.60921
5600	3.74819	2.49879	8250	3.91645	2.61097
5650	3.75205	2.50137	8300	3.91908	2.61272
5700	3.75587	2.50392	8350	3.92169	2.61446
5750	3.75967	2.50645	8400	3.92428	2.61619
5800	3.76343	2.50895	8450	3.92686	2.61790
5850	3.76716	2.51144	8500	3.92942	2.61961
5900	3.77085	2.51390	8550	3.93197	2.62131
5950	3.77452	2.51634	8600	3.93450	2.62300
6000	3.77815	2.51877	8650	3.93702	2.62468
6050	3.78176	2.52117	8700	3.93952	2.62635
6100	3.78533	2.52355	8750	3.94201	2.62801
6150	3.78888	2.52592	8800	3.94448	2.62966
6200	3.79239	2.52826	8850	3.94694	2.63130
6250	3.79588	2.53059	8900	3.94939	2.63293
6300	3.79934	2.53289	8950	3.95182	2.63455
6350	3.80277	2.53518	9000	3.95424	2.63616
6400	3.80618	2.53745	9050	3.95665	2.63777
6450	3.80956	2.53971	9100	3.95904	2.63936
6500	3.81291	2.54194	9150	3.96142	2.64095
6550	3.81624	2.54416	9200	3.96379	2.64253
6600	3.81954	2.54636	9250	3 96614	2.64409
6650	3.82282	2.54855	9300	3.96848	2.64566
6700	3.82607	2.55072	9350	3.97081	2.64721
6750	3.82930	2.55287	9400	3.97313	2.64875
6800	3.83251	2.55501	9450	3.97543	2.65029
6850	3.83569	2.55713	9500	3.97772	2.65182
6900	3.83885	2.55923	9550	3.98000	2.65334
6950	3.84198	2.56132	9600	3.98227	2.65485
7000	3.84510	2.56340	9650	3.98453	2.65635
7050	3.84819	2.56546	9700	3.98677	2.65785
7100	3.85126	2.56751	9750	3.98900	2.65934
7150	3.85431	2.56954	9800	3.99123	2.66082
7200	3.85733	2.57155	9850	3.99344	2.66226
7250	3.86034	2.57356	9900	3.99564	2.66376
7300	3.86332	2.57555	9950	3.99782	2.66522
7350	3.86629	2.57752	10000	4.00000	2.66667
7400	3.86923	2.57949	10050	4.00217	2.66811
7450	3.87216	2.58140	10100	4.00432	2.66955
7500	3.87506	2.58337	10150	4.00647	2.67098
7550	3.87795	2.58530	10200	4.00860	2.67240
7600	3.88081	2.58721	10250	4.01072	2.67382
7650	3.88366	2.58911	10300	4.01284	2.67522
7700	3.88649	2.59099	10350	4.01494	2.67663
7750	3.88930	2.59287	10400	4.01703	2.67802
7800	3.89209	2.59473	10450	4.01912	2.67941
7850	3.89487	2.59658	10500	4.02119	2.68079
7900	3.89763	2.59842	10550	4.02325	2.68217

TABLE B.

VALUES OF *V. C.* AND *Log. C.*

1 Speed *V.* Knots.	2 $C=\frac{H\cdot P}{D^{\frac{2}{3}}}$	3 *Log. C.*	1 Speed *V.* Knots.	2 $C=\frac{H\cdot P}{D^{\frac{2}{3}}}$	3 *Log. C.*
10.0	6.24	0.79518	15.0	18.14	1.25864
.1	6.39	0.80550	.1	18.47	1.26647
.2	6.55	0.81624	.2	18.80	1.27416
.3	6.72	0.82737	.3	19.13	1.28172
.4	6.90	0.83835	.4	19.46	1.28914
.5	7.07	0.84942	.5	19.89	1.29863
.6	7.24	0.85974	.6	20.12	1.30363
.7	7.42	0.87040	.7	20.48	1.31133
.8	7.60	0.88081	.8	20.84	1.31890
.9	7.77	0.89042	.9	21.25	1.32736
11.0	7.94	0.89982	16.0	21.67	1.33586
.1	8.13	0.91009	.1	22.03	1.34301
.2	8.32	0.92012	.2	22.40	1.35025
.3	8.55	0.93197	.3	22.75	1.35698
.4	8.78	0.94349	.4	23.10	1.36361
.5	8.97	0.95279	.5	23.52	1.37144
.6	9.16	0.96190	.6	23.94	1.37912
.7	9.38	0.97220	.7	24.36	1.38668
.8	9.60	0.98227	.8	24.74	1.39340
.9	9.78	0.99034	.9	25.19	1.40123
12.0	9.96	0.99826	17.0	25.65	1.40909
.1	10.22	1.00945	.1	26.05	1.41581
.2	10.48	1.02036	.2	26.46	1.42260
.3	10.67	1.02816	.3	26.90	1.42975
.4	10.90	1.03743	.4	27.34	1.43680
.5	11.14	1.04689	.5	27.78	1.44373
.6	11.38	1.05614	.6	28.22	1.45056
.7	11.59	1.06408	.7	28.66	1.45728
.8	11.80	1.07188	.8	29.10	1.46389
.9	12.05	1.08099	.9	29.60	1.47129
13.0	12.31	1.09026	18.0	30.11	1.47857
.1	12.58	1.09968	.1	30.52	1.48458
.2	12.85	1.10890	.2	30.94	1.49052
.3	13.17	1.11959	.3	31.47	1.49790
.4	13.40	1.12710	.4	32.00	1.50515
.5	13.66	1.13545	.5	32.48	1.51162
.6	13.92	1.14364	.6	32.96	1.51799
.7	14.19	1.15198	.7	33.45	1.52440
.8	14.46	1.16017	.8	33.94	1.53071
.9	14.74	1.16850	.9	34.51	1.53795
14.0	15.03	1.17696	19.0	35.09	1.54518
.1	15.33	1.18554	.1	35.59	1.55133
.2	15.64	1.19424	.2	36.10	1.55751
.3	15.92	1.20194	.3	36.62	1.56372
.4	16.20	1.20952	.4	37.12	1.56961
.5	16.54	1.21854	.5	37.69	1.57623
.6	16.88	1.22737	.6	38.26	1.58275
.7	17.18	1.23502	.7	38.83	1 58917
.8	17.48	1.24254	.8	39.40	1.59550
.9	17.81	1.25066	.9	39.99	1.60195

TABLE B.—Continued.

1 Speed V. Knots.	2 $C=\frac{H.P}{D^{\frac{2}{3}}}$	3 *Log. C.*	1 Speed V. Knots.	2 $C=\frac{H.P}{D^{\frac{2}{3}}}$	3 *Log. C.*
20.0	40.58	1.60831	22.75	58.60	1.76790
20.25	41.95	1.62273	23.0	60.51	1.78183
20.5	43.50	1.63849	23.25	62.45	1.79553
20.75	45.00	1.65321	23.5	64.40	1.80889
21.0	46.60	1.66839	23.75	66.35	1.82184
21.25	48.20	1.68305	24.0	68.42	1.83518
21.5	49.70	1.69636	24.25	70.50	1.84819
21.75	51.40	1.71096	24.5	72.52	1.86046
22.0	53.25	1.72632	24.75	74.70	1.87332
22.25	54.90	1.73957	25.0	77.00	1.88649
22.5	56.80	1.75435			

Extract from Specifications for Tests of a 15,000,000 Gallon Pumping Engine for the City of Buffalo, N. Y., 1883.

BY D. M. GREENE.

The pumping machinery, after it shall have been fully completed and adjusted by the contractor, and after the contractor shall have notified the Water Commissioners of its entire completion and readiness for service, will be subjected by the latter, to the following tests:

1. A *capacity* test of 24 hours' duration.

2. At the conclusion of the capacity test, and without stopping the machinery, a *duty* test will be begun, which shall continue 48 hours.

3. Subsequently, and within two weeks after the conclusion of the tests above specified, a working test shall be begun; provided the engine goes through the first two tests satisfactorily. The working test shall consist of runs aggregating 1,800 hours, and shall be completed within the limits of three months from the commencement of the test.

During the term of the working test the contractor is to furnish an engineer to assist in conducting the same, whose compensation, to be paid by the Water Commissioners, shall be the same as that of the engineer furnished by the water department. The engineer will be allowed to make such slight

repairs as are customary among engineers, without any record of the same; but all material or outside work, by whomsoever furnished, will be charged to the contractor.

In all of these tests, good merchantable Anthracite coal will be used; it will be furnished by the Water Commissioners, but may be selected by the contractor.

In making the second, or first *duty* test, the fires shall, at the beginning of the test, be in their normal condition, with one furnace ready for firing; the other furnaces having been previously fired, in their proper order, and at proper intervals of from 20 to 30 minutes, respectively. Reasonably uniform intervals shall be observed in the firing throughout the test; and, at its conclusion, the fires shall be in the same condition, as to thickness and degree of exhaustion, as at the beginning; *i. e.*, one furnace shall be ready for firing, and the same intervals of time shall have elapsed, since the last previous firing of each of the other furnaces, as had elapsed since the last previous firing at the beginning of the test, respectively.

The hourly rate of coal consumption, and the rate at which the coal is supplied to the furnaces, shall be kept as nearly uniform as practicable throughout the time occupied by both the capacity and first duty tests. The mean quantity of coal put into the furnaces per minute, during the time occupied in making a duty test, will be regarded as the coal consumption (C) in determining the duty.

The net work of the pumps, or the product of the mean load, in pounds, under which they are worked, by the mean space traversed by their plungers, in feet per minute, will be regarded and treated as the number of foot-pounds of work due to the mean coal consumption C.

The effective areas of the pump-plungers will be taken as the areas due to their diameters, less one-half the areas due to the diameters of their pump-rods, respectively; unless the pump-rods shall pass through both ends of the pump-chambers; in which case the entire area of the sections of the pump-rods will be deducted from the areas due to the diameters of the plungers.

Let the *aggregate* of the effective areas of the plungers of any machine, in square inches, be represented by A.

The mean load, or pressure, in pounds per square inch (P),

upon the effective area of a pump-plunger, in any case, will be determined as follows:

The mean pressure, as indicated by the pressure gauge on the force main or delivery pipe being p, and the difference in level, between the centre of said pressure gauge and the mean level of the surface of the water in the pump-well, in feet, divided by 2.3 being p', the mean load or pressure per square inch, in pounds, will be

$$P = p + p' + 1.$$

One pound, or 2.3 feet, being added to cover the effects of friction through the suction pipe and pumps, and of temperature. No other allowance, of any kind whatever, or for any purpose whatever, will be made.

During the 24-hour capacity test, and during the 48 hour duty test, the steam and water pressures and the readings of the engine counter will be noted and recorded every 15 minutes; while the quantity of coal used, and the height of the water in pump-well, will be recorded every hour.

During the working duty test, the steam and water pressures and the reading of the engine-counter, will be noted and recorded every 30 minutes, and the quantity of coal used—and the level of the water in the pump-well—every hour.

In making the working test, the same precautions, as to thickness and conditions of fires, shall be observed as are prescribed for the beginning of the first duty test; and thereafter, all coal consumed, during the three months test, shall be charged to the engine, excepting such coal as may be used for banking fires. In case the operation of the machinery shall be interrupted for any reason by the water department, the coal lost, and the coal required to start the engine again, shall not be charged to the engine.

The capacities of the pumps will be estimated by the following formulæ, prepared in conformity with the foregoing, and in which the efficiency of the pumps is taken at 0.97. In other words, three per cent. is allowed for loss of action due to the probable imperfect filling of the pumps.

For single acting pumps,

$$Q = 72.561\ A\ N\ S.$$

For double acting pumps,

$$Q = 145.122\ A\ N\ S.$$

The *duties* will be calculated by the following formulæ:
If the pumps be single acting,

$$D = \frac{100\,A\,P\,N\,S}{C}.$$

If the pumps be double acting,

$$D = \frac{200\,A\,P\,N\,S}{C}.$$

In the foregoing formulæ, as already explained,

A = the sum of the effective areas of all the pump-plungers, of any engine, in square inches.

P = the common net load, or pressure, in pounds per square inch, upon the pump-plungers—to be deduced from the means of the recorded data.

S = the common stroke of the pumps of any engine, in feet.

N = the mean number of revolutions of the engine, or the mean number of double strokes of the pumps, per minute.

Q = the pumping capacity of the engine, in U. S. gallons, in 24 hours.

D = the *duty*, in foot-pounds, per hundred pounds of coal.

The capacity and duty tests of the engine will be made by an expert, who shall be selected by the Water Commissioners, subject to the approval of the contractor; or, in the event of the failure of the Water Commissioners to select an expert who shall be satisfactory to the contractor, the Water Commissioners shall select one expert, the contractor a second, and the two experts thus selected shall select a third. The expense, for compensation of the experts, shall be divided equally between the water board and the contractor.

In making the first and second tests, no expert is to be engaged who may be interested in business with the contractor, or who is in the employment of either the contractor or water board.

The working test shall be made by the chief engineer of the water department, and the engineer furnished by the contractor. In conducting the working tests, the boilers and engines must be run in the general and usual manner of running pumping engines at their daily ordinary duty.

The conclusions of the expert, or of the board of experts and engineers, shall be conclusive and binding upon both parties.

The engine, boiler, chimney and foundations furnished, together with all attachments, appliances and connections, must be strictly first-class, in material, design, workmanship, finish and in operation.

It must be completed, delivered, set up, adjusted, and in readiness for use, on or before the expiration of the time named in the proposal or contract, and payment for the same will be made within thirty days after the completion of the specified plant, in accordance with the foregoing specifications, and in accordance with such other and additional general or detailed specifications as may be incorporated in, and form a part of the contract to be made, and after the tests and acceptance of the same by the Water Commissioners.

Tenders will be received at the Water Commissioners' office up to the eighth day of November next, at three o'clock in the afternoon.